EE5585: HOMEWORK 2

ALL PROBLEMS CARRY EQUAL POINTS

(1) Design a 3-bit uniform quantizer (by specifying the decision boundaries and reconstruction levels) for a source with the following pdf:

$$f_X(x) = \frac{1}{6}e^{-\frac{|x|}{3}}.$$

(2) What is the mean and variance of the random variable of Problem (1) above? Derive the mean of the output of uniform quantizer you designed for the above problem. What is the mean of the optimum quantizer for this distribution?

(3) Consider the following compression scheme for binary sequences. We divide the binary sequences into blocks of size 16. For each block if the number of zeros is greater than or equal to 8 we store a 0, otherwise we store a 1. If the sequence is random with probability of zero 0.9, compute the rate and average distortion (Hamming metric). Compare your result with the corresponding value of rate distortion function for binary sources.

(4) Let X be zero mean and σ^2 variance Gaussian random variable. That is

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}$$

and let the distortion measure be squared error. Find the optimum reproduction points for 1-bit scalar quantization and the expected distortion for 1-bit quantization. Compare this with the rate-distortion function of a Gaussian random variable.

(5) Consider a source X uniformly distributed on the set $\{1, 2, \ldots, m\}$. Find the rate distortion function for this source with Hamming distortion; that is, $d(x, \hat{x}) = 0$ when $x = \hat{x}$ and $d(x, \hat{x}) = 1$ when $x \neq \hat{x}$.

(6) Let $\{X\}_{i=0}^{\infty}$ be an i.i.d. binary sequence with probability of 1 being 0.3. Calculate $F(01110) = P(0.X_1X_2X_3X_4X_5 < 0.01110)$. How many bits of $F = 0.F_1F_2\ldots$ can be known for sure if it is not known how the sequence $0.01110\ldots$ continues?