EE5585 Data Compression March 26, 2013

Lecture 16

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1 Review of Homework 6

The review is omitted from this note.

2 Linde-Buzo-Gray (LBG) Algorithm

Let us start with an example of height/weight data.

H (in)	W (lb)
65	170
70	170
56	130
80	203
50	153
76	169

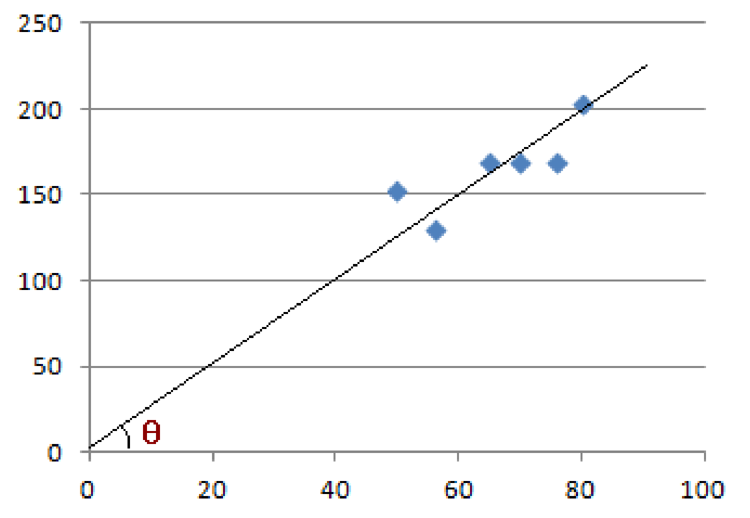

Figure 1:

We need to find a line such that most points are around this line. This means the distances from the points to the line is very small. Suppose $W = 2.5H$ is the equation of the straight line. Let A be a matrix to project values to this line and its perpendicular direction.

We have

$$A = \begin{bmatrix} 0.37 & 0.92 \\ -0.92 & 0.37 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$X = \begin{bmatrix} 65 \\ 170 \end{bmatrix}, \begin{bmatrix} 70 \\ 170 \end{bmatrix}, \begin{bmatrix} 56 \\ 130 \end{bmatrix} \ldots$$

Then by rotating x-axis, we have a new axis:

$$Y = AX$$

From the above equation, we can get a set of new H-W table as following:

H (new)	W (new)
182	3
184	-2
191	-4
218	1
161	10
181	-9

We can get back original data by rotating the axis again: $X = A^{-1}Y$, for example,

$$\begin{bmatrix} 65 \\ 170 \end{bmatrix} = A^{-1} \begin{bmatrix} 182 \\ 3 \end{bmatrix}$$

where,,

$$A^{-1} = \begin{bmatrix} 0.37 & -0.92 \\ 0.92 & 0.37 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} = A^T.$$

Note,

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix} * \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Now, neglect the right column of the new table and set them to 0, that is:

H (new)	W (new)
182	0
184	0
191	0
218	0
161	0
181	0

Multiplying each row by A^T we have,

H	W
68	169
68	171
53	131
81	201
60	151
67	168

Compare this table with the original table. We find that the variations are not large. So this new table can be used.

3 Transform Coding

1. For an orthonormal transformation matrix A,

$$A^T = A^{-1} \Leftrightarrow A^T A = AA^T = I. \tag{1}$$

Suppose $y = Ax, \hat{x} = A^{-1}\hat{y}$, where $\hat{y}$ is the stored/compressed value. We introduce error

$$= \; \| \, y - \hat{y} \, \|_2^2 \tag{2}$$
$$= \; \| \, Ax - A\hat{x} \, \|_2^2 \tag{3}$$
$$= \; \| \, A(x - \hat{x}) \, \|_2^2 \tag{4}$$
$$= \; [A(x - \hat{x}]^T * [Ax - \hat{x}] \tag{5}$$
$$= \; (x - \hat{x})^T * A^T * A(x - \hat{x}) \tag{6}$$
$$= \; (x - \hat{x})^T * (x - \hat{x}) \tag{7}$$
$$= \; \| \, x - \hat{x} \, \|_2^2 \tag{8}$$

If we do not introduce a lot of errors in y, then there won't be a lot of error for x.

2.

$$E[y^T y] = E[x^T A T A x] = E[x^T x]. \tag{9}$$

which means the input and output energies are the same. This is known as Parseval's identity.

4 Hadamard Transform

Suppose,

$$A = 1/\sqrt{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{10}$$

-1 is shaded as black in the figure below.:

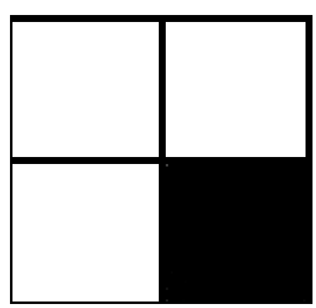

Figure 2:

Use Kronecher Product to define: $A_4 = A_2 \oplus A_2$ we have

$$A_4 = 1/\sqrt{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

The Kronecker product is defined to be,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \otimes \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$= \begin{bmatrix} b_{11}M & b_{12}M \\ b_{21}M & b_{22}M \end{bmatrix}$$

where,

$$M = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

A_4 can be represented as below.

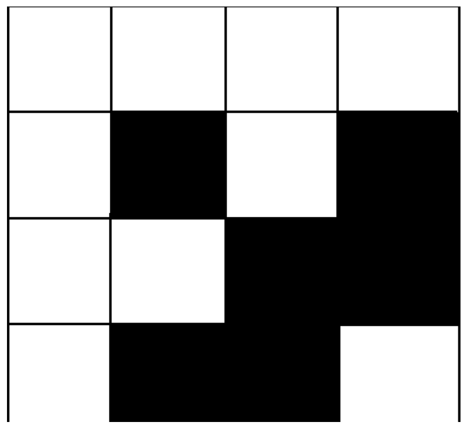

Figure 3:

Then we can have $A_{16} = A_4 \oplus A_4$ in this similar way.

5 Open Problem: Fix-free code, Conjectures, Update-efficiency

Instructor's note: This section is removed (the reason can be explained in person). Please consult your notes.