EE5585: MIDTERM EXAMINATION

FEB 26, 2013 11:15AM - 12:30 PM

Books/notes/electronic calculators: permitted.
Cellular/mobile phones, computers, and internet: strictly prohibited.
Make reasonable assumptions to solve the problems if you think important information is missing.

(1) Suppose a binary file of size (length) 10000 needs to be compressed such that it can be recovered with at most 5% Hamming distortion. What do you think is the best rate of compression? Reason. 2

(2) Consider a two dimensional unit square with corners $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$. There are four quantization points $(1/4,1/4)$, $(1/4,3/4)$, $(3/4,1/4)$, $(3/4,3/4)$. That is, any point inside the square can be compressed to 2 bits by mapping to any of these four quantization points (see figure below). Suppose a point is randomly and uniformly picked up from this unit square. What is the probability that upon compression the Euclidean distortion is greater than $1/4$? What is the worst case distortion? 3

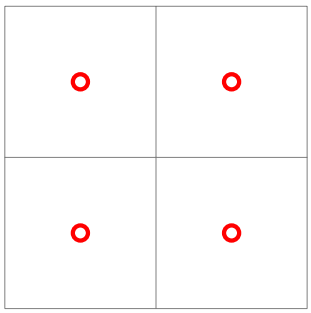

FIGURE 1. Problem 2

(3) Find a set of lengths $(l_1, l_2, \dots)$ of binary codewords of a prefix code that minimizes $\sum_i p_i l_i$ for the following probability distribution

$$P = \left\{ \frac{9}{10}, \frac{9}{10} \cdot \frac{1}{10}, \frac{9}{10} \cdot \left(\frac{1}{10}\right)^2, \frac{9}{10} \cdot \left(\frac{1}{10}\right)^3, \frac{9}{10} \cdot \left(\frac{1}{10}\right)^4, \dots \right\}$$

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(4) Suppose you are given a binary file that is a string of 5050 zeros (i.e., $00\ldots0$).

(a) What would be the the number of distinct sequences in the Lempel Ziv parsing? 2

(b) What will be the length of the compressed sequence after Lempel Ziv encoding? 2

(c) Now suppose a file of $\frac{m(m+1)}{2}$ consecutive 0's is given. What is the rate of compression achievable by Lempel Ziv algorithm (in terms of m)? 3

(5) An i.i.d. binary-$\{0,1\}$ source is such that $\Pr(0) = 2/3$.

(a) What is a Huffman code for this source (Hint: club multiple positions together)? With your code, how far away is the average length of codewords from entropy? 4

(b) Can you use Shannon code for this problem? If yes, what would be the average length for that? 4