Summary of Expressions

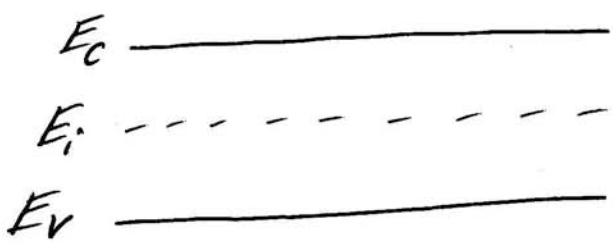
$$n = N_c e^{-(E_c - E_f)/kT} = n_i e^{(E_f - E_i)/kT}$$

$$p = N_v e^{-(E_f - E_v)/kT} = n_i e^{(E_i - E_f)/kT}$$

$$np = n_i^2 \qquad n_i = N_c e^{-(E_c - E_i)/kT} = N_v e^{-(E_i - E_v)/kT}$$

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \qquad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

Example Find the position of the intrinsic Fermi Level, E_i, in silicon at $T = 300K$



E_i is the fermi level when a material is undoped.

In silicon $n_i = 1.1 \times 10^{10} cm^{-3}$ at $T = 300K$

since $n = N_c e^{-(E_c - E_F)/kT}$ and $p = N_v e^{-(E_F - E_v)/kT}$

then if $n = p = n_i$,

$n_i = N_c e^{-(E_c - E_i)/kT}$ and $n_i = N_v e^{-(E_i - E_v)/kT}$

$$\frac{n_i}{N_c} = e^{-(E_c - E_i)/kT}$$

$$\ln \frac{n_i}{N_c} = -\frac{(E_c - E_i)}{kT}$$

$$E_c - E_i = +kT \ln \frac{N_c}{n_i} \qquad N_c = 2.8 \times 10^{19}\ cm^{-3}$$

$$\boxed{E_c - E_i = .563 eV}$$

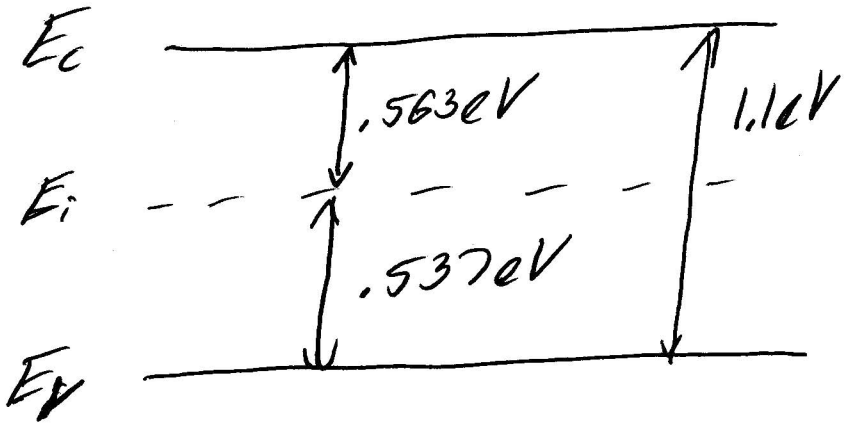
Similarly $E_i - E_v = kT \ln \frac{N_v}{n_i}$ $\qquad N_v = 1.04 \times 10^{19}\ cm^{-3}$

$$\boxed{E_i - E_v = .537}$$

Note that

$$(E_c - E_i) + (E_i - E_v) = E_c - E_v$$

$$= E_{gap} = 1.1 eV \checkmark$$



Intrinsic Silicon

Another way to find E_i is:

$$n_i = N_c e^{-(E_c - E_i)/kT} = N_v e^{-(E_i - E_v)/kT}$$

divide through by N_v

$$\frac{N_c}{N_v} e^{E_i/kT} e^{-E_c/kT} = e^{E_v/kT} e^{-E_i/kT}$$

$$e^{2E_i/kT} = \frac{N_v}{N_c} e^{(E_c+E_v)/kT}$$

take ln of both sides

$$\frac{2E_i}{kT} = \ln\frac{N_v}{N_c} + \frac{E_c+E_v}{kT}$$

$$E_f = \underbrace{\frac{E_c + E_v}{2}}_{\text{midgap}} + \frac{kT}{2} \ln \frac{N_v}{N_c}$$

Recall

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$\therefore E_f = \frac{E_c + E_v}{2} + \frac{kT}{2} \ln\left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

$$\boxed{E_i = \frac{E_c + E_v}{2} + \frac{3kT}{4} \ln \frac{m_p^*}{m_n^*}}$$

For silicon at $T = 300K$ (Let $E_v = 0$)

$kT = .026eV$

$m_p^* = .56 m_0$

$m_n^* = 1.08 m_0$

$\frac{E_c + E_v}{2} = .55$

$$\boxed{E_i = .537eV \quad \text{same as before!}}$$