Due: Friday, 12/13/13 at 5pm in Xiaofan's office

Problem 1: An inverted pendulum (of mass m and length $2l$) on a cart (of mass M) is shown in Figure 1. The linearized model of this system is given by:

$$\begin{aligned} \ddot{s} &= -\frac{mg}{M}\theta + \frac{1}{M}u, \\ \ddot{\theta} &= \frac{(M+m)g}{2Ml}\theta - \frac{1}{2Ml}u, \end{aligned}$$

where s is the cart position, θ is the pendulum angle, and u is the force applied on a cart. The system parameters have the following values: $M = m = 2l = 1$ and $g = 9.8$.

a) Determine the state equation of this system with: $x_1 = s$, $x_2 = \theta$, $x_3 = \dot{s}$, $x_4 = \dot{\theta}$.

b) Check controllability from the input which is the force on the cart.

c) Check observability in the following two cases:

- when the measured output is the cart position,
- when the measured output is the pendulum angle.

d) Based on the test that you performed in part c) above, if you could only measure either cart position or pendulum angle, which one would you choose?

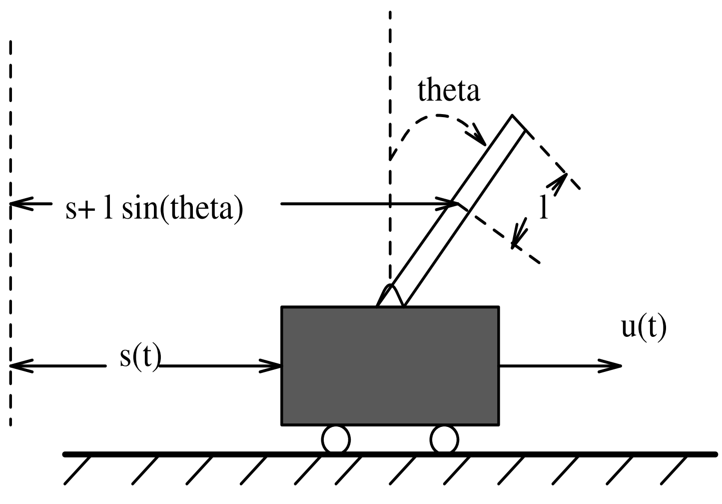


Figure 1: Inverted pendulum on a cart.

Problem 2: For a system from **Problem 1** design two observer based controllers for the following two situations:

a) The measured output is the cart position.

b) The measured output is **both** cart position **and** pendulum angle.

In both of the above designs, use LQR design to find K and L to stabilize matrices $A - BK$ and $A - LC$. Use $Q = I$ and $R = 1$ (in the case of two measurements, use $R = I$, the 2×2 identity matrix).

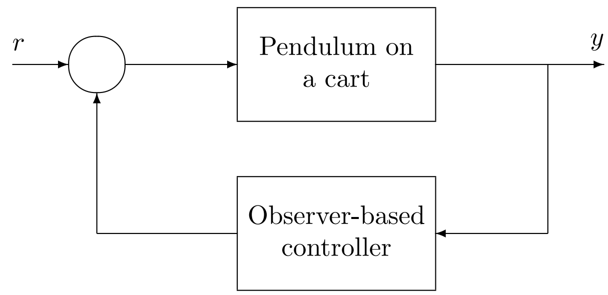
Note: you will use the same K for both controllers, the only difference between them will be L since for each case, the C matrix of the plant is different. Pay attention to the dimension of matrices, and use the MATLAB command lqr in this problem.

Problem 3: As a check for your design in **Problem 2**, do the following (do this only for the case of the controller that measures cart position):

a) Find the transfer function of the linearized pendulum from cart force input to cart position output. You can use the MATLAB commands `ss2tf` or `tfdata` for this. Note the degree of this transfer function.

b) Find the transfer function of the observer based controller. Note the degree of this transfer function.

c) Connect the two transfer functions in feedback as shown in the diagram below, and compute the transfer function from r to y. Note the degree of this transfer function. Is it stable?



Note: you can use the MATLAB command `feedback` to solve this problem. Since this command assumes negative feedback (and here the negative sign is already included in the controller), you will need to multiply the system in the feedback loop with -1 when using this command. If you are unsure how this command works type `help feedback`, and check it first on a simple example for which you can easily determine the answer by hands.

d) Find the poles of the closed-loop transfer function. Check that they are exactly the eigenvalues of $A - BK$ and $A - LC$.

Note: to check stability, you can use the MATLAB function `roots` to find the roots of a polynomial.

Problem 4: Simulate the pendulum on a cart in feedback with three different controllers (from initial conditions which are all zeros except $\theta(0) = 10^o$):

a) A state feedback controller using K designed with LQR (use the K designed for the observer based controller).

b) Observer based controller measuring both cart position and pendulum angle.

c) Observer based controller measuring only cart position.

Produce two separate graphs, one for cart position and one for pendulum angle. On each of the graphs plot and compare the responses of the three different controllers. Discuss your observations.

Note: you can use the MATLAB command `initial` in this problem.