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Lecture 1

What is this course about?

- linear dynamical systems

↳ $\frac{dx(t)}{dt} = A(t)\,x(t) + B(t)\cdot u(t)$ *

$y(t) = C(t)\cdot x(t) + D(t)\cdot u(t)$

where: t - time

d/dt - derivative wrt time

$x(t)$ - state, $u(t)$ - input, $y(t)$ - output } vectors: $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$

→ meaning $x(t) \in \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}$

$x_i(t) \to$ scalar

$A(t)$, $B(t)$, $C(t)$, $D(t)$ - real matrices of corresponding dimensions (details later gator)

ex// $A(t) \in \mathbb{R}^{n\times n}$

$B(t) \in \mathbb{R}^{n\times m}$ ⟶ n rows, m columns

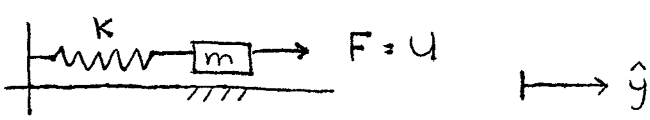
$C(t) \in \mathbb{R}^{p\times n}$

$D(t) \in \mathbb{R}^{p\times m}$

In discrete time: $x(t+1) = A(t)\cdot x(t) + B(t)\cdot u(t)$

$y(t) = C(t)\cdot x(t) + D(t)\cdot u(t)$

ex//



input: force u

output: position y

$m\cdot \frac{d^2y(t)}{dt^2} + k\,y(t) = u(t)$ — does this fit into form above? *

no! we can only differentiate once!

introduce $x_1(t) = y(t)$

$x_2(t) = dy(t)/dt = \dot{y}(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} \cdot u(t)$$

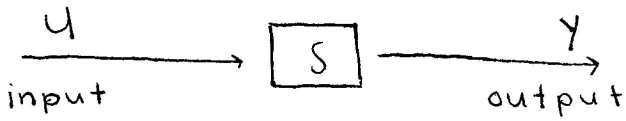
$$u(t) = [1, 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0\cdot u(t)$$

why bother? It's everywhere!

A rather general view of a system

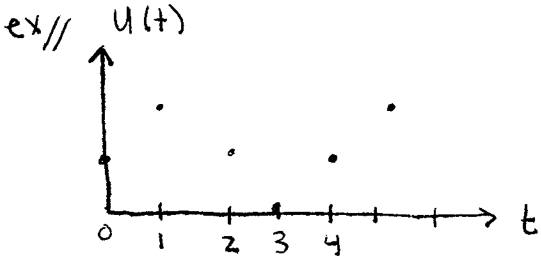
system: mapping from input to output signals

input signal $\xrightarrow{\text{system}}$ output signal

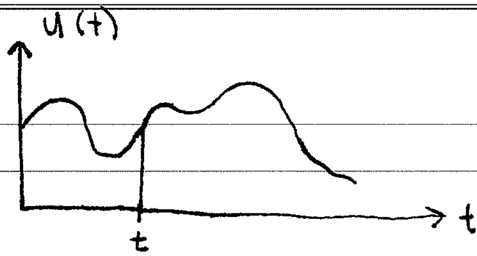


signal - function of time

ex//



discrete $t = \{0, 1, 2, \ldots\}$



continuous $t \in [0, m]$

$t \in [0, \infty)$

$t \in (-\infty, \infty)$

$y = Su$ $\qquad y(t) = [Su](t)$

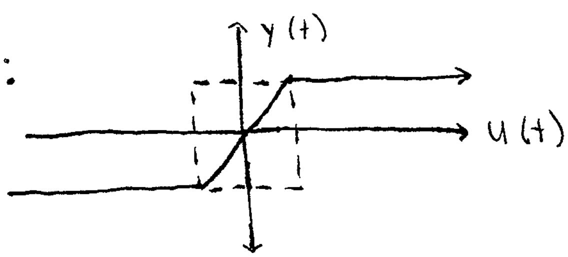
Ex 1: MS discussed earlier

* need to know initial condition!
eternal state

$m \cdot \ddot{y}(t) + k \cdot y(t) = u(t)$

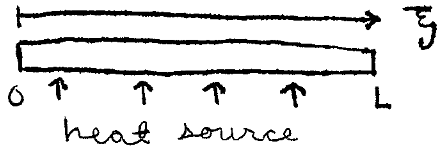
→ in addition to $u(t)$, need to know $(y(0), \dot{y}(0))$ in order to determine time evolution of output y

Ex 2:



$$y(t) = \text{sat}(u(t)) = \begin{cases} +1 & u(t) > 1 \\ u(t) & -1 \le u(t) \le +1 \\ -1 & u(t) < -1 \end{cases}$$

Ex 3:



$$\partial T(t, \xi)/\partial t = \partial^2 T(t, \xi)/\partial \xi^2 + u(t, \xi)$$

↳ an example of an infinite dimensional system

↱ $x(t) = T(t, \cdot)$

system: $x(t)$ function of any fixed time
(not a vector w/ fixed # of components)

finite dimensional approximation of this system can be brought into state space form

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System Properties

1. linearity
2. time invariance
3. Causality
4. memory
5. finite vs infinite dimensional