Lecture 18

4/8/2014

Backstepping

$$\dot{x} = f(x) + g(x) \cdot \xi$$
$$\dot{\xi} = u$$

$x(t) \in \mathbb{R}^n$

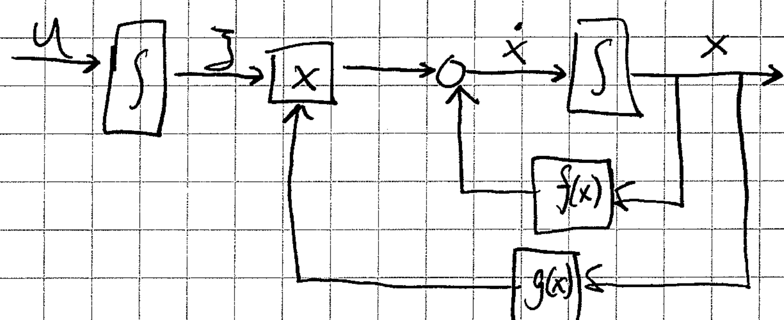
$f(x), g(x)$: vector fields

$\xi(t) \in \mathbb{R}$: scalar

$u(t) \in \mathbb{R}$: scalar

Objective: Find $u = u(x, \xi)$ (ie. state feedback)

S.T. $(\bar{x}, \bar{\xi}) = (0,0)$ is G.A.S



A1 Assumption: $\xi = \alpha(x)$ there is

$V_1(x)$ S.T. $\dot{V}_1(x) = \frac{\partial V}{\partial x} [f(x) + g(x)\alpha(x)]$

$= -W_1(x) < 0$

(A1) $\Rightarrow$ $\bar{x} = 0$ is GAS Eq Pt. of

$$\dot{x} = f(x) + g(x)\,\alpha(x)$$

Problem: ξ not Control
(it's a State Variable)

Solution: Backstepping

Step:
$$z = \xi - \alpha_1(x)$$
$$\dot{z} = \dot{\xi} - \dot{\alpha}_1 =$$
$$= u - \frac{\partial \alpha_1}{\partial x}\dot{x} =$$
$$= u - \frac{\partial \alpha_1}{\partial x}\Big[f(x) + \underbrace{g(x)\,\alpha_1 + g(x)\,z}_{g(x)\cdot\xi}\Big]$$

Augment $V_1(x)$ w/ $\frac{1}{2}(\xi - \alpha_1(x))^2 = \frac{1}{2}z^2$

$$\Rightarrow V_2(x,z) = V_1(x) + \frac{1}{2}z^2$$

$$\dot{V}_2 = \dot{V}_1 + z\dot{z} =$$
$$= \underbrace{\frac{\partial V_1}{\partial x}\big[f(x) + g(x)\,\alpha_1(x)\big]}_{\text{well behaved}} + \frac{\partial V_1}{\partial x}\big[g(x)z\big] + z(u - \dot{\alpha}_1)$$
$$= -W_1(x) + z\Big(u - \dot{\alpha}_1 + \frac{\partial V_1}{\partial x}g(x)\Big)$$

A choice of u:

$$u = -k \cdot z + \dot{\alpha}_1 - \frac{\partial V_1}{\partial x} g(x)$$

gives

$$\dot{V}_2 = -W_1(x) - kz^2 < 0 \quad , \quad k > 0$$