Due Tu 04/12/16

1. A simplified model of an axial compressor, used in jet engine control studies, is given by the following second order system

$$\begin{aligned}\dot{\phi} &= -\frac{3}{2}\phi^2 - \frac{1}{2}\phi^3 - \psi \\ \dot{\psi} &= \frac{1}{\beta^2}(\phi + 1 - u).\end{aligned}$$

   This model captures the main surge instability between the mass flow and the pressure rise. Here, ϕ and ψ are deviations of the mass flow and the pressure rise from their set points, the control input u is the flow through the throttle, and β is positive constant.

   (a) Use backstepping to obtain a control law that stabilizes the origin $(\phi, \psi) = 0$.

   (b) Use Sontag's Formula and the Control Lyapunov Function obtained in part (a) to obtain an alternative control law.

2. Consider the mass-spring-damper system described by

$$m\ddot{y} + \beta\dot{y} + ky = u,$$

   (a) If $y(t)$ and $u(t)$ are available for measurement, design a gradient algorithm to estimate constant but unknown parameters m, β, and k.

   (b) Simulate your algorithm in (a) assuming that true values are $m = 20$, $\beta = 0.1$, and $k = 5$. Repeat your simulation for different choices of $u(t)$ and observe the resulting parameter convergence properties.

3. Consider the reference model:

$$\dot{y}_m = -ay_m + r(t), \qquad a > 0,$$

   and the plant:

$$\dot{y} = a^\star y + b^\star u, \qquad b^\star \neq 0.$$

   (a) Show that a controller of the form:

$$u = \theta_1 y + \theta_2 r(t)$$

   with an appropriate choice of gains $\theta_1^\star$ and $\theta_2^\star$, drives the tracking error $e := y - y_m$ asymptotically to zero.

   (b) Now suppose $a^\star$ and $b^\star$ are unknown parameters, but the sign of $b^\star$ is known. Show that the adaptive implementation of the controller above achieves tracking when the gains are updated according to the rule:

$$\dot{\theta}_1 = -\text{sign}(b^\star)\gamma_1 ye, \qquad \dot{\theta}_2 = -\text{sign}(b^\star)\gamma_2 re,$$

   where $\gamma_1 > 0$ and $\gamma_2 > 0$.

   (c) Provide a condition that also guarantees $\theta_1(t) \to \theta_1^\star$ and $\theta_2(t) \to \theta_2^\star$ as $t \to \infty$.