HW#2

Show all work for full credit!

1. Let $G_1(s) = \frac{s-1}{s+1}$ and $G_2(s) = \frac{s+3}{(s-1)(s+2)}$.
   (a) Obtain state space realizations of G_1 and G_2 such that the corresponding state dimension is equal to the degree of the denominator of the transfer function.
   (b) Determine the transfer function $G = G_1(s)G_2(s)$ and determine the induced realization obtained from the realizations of G_1 and G_2 obtained in the question above
   (c) Determine a realization of G such that the state dimension is equal to the degree of the denominator of the transfer function G.

2. Determine the state space representation of a generalized feedback interconnection from the state space realization of individual components (see course notes for the description of generalized feedback interconnection).

3. Let $A = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = 0$ be a state space realization. Determine the transfer function. Determine e^{At} and determine the step response of the state from the variation of parameters formula and the output from the transfer function.

4. Let $V = \mathbb{R}^3$ be the vector space (with regular vector addition and scalar multiplication on real scalars). Then show that $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 8 \end{bmatrix} \right\}$ is a linearly independent set. Show that $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$ is a linearly dependent set.

5. Let $A \in \mathbb{R}^{n \times n}$. Show that the set $\mathcal{V} = \{x(t) \text{ such that } \dot{x}(t) = Ax(t)\}$ is a vector space. The $(+)$ and $(\cdot)$ operators are respectively defined by $(x_1 + x_2)(t) = x_1(t) + x_2(t)$ and $(\alpha.x)(t) = \alpha x(t)$. Assume the scalar field to be real numbers.

6. Suppose $\{w_1, w_2, \ldots, w_n\}$ is a linearly dependent set. Then show that one of the vectors must be a linear combination of the others.