Frequency domain specifications in \(sL(s)\) small in \([0, \omega_b]\) leads to \(I_{II}\) being large in \([0, \omega_b]\)
- \(T(s)\) small in \([\omega_b, \infty)\) leads to \(I_{II}\) being small in \([\omega_b, \infty)\)

- Sensitivity transfer function \(\frac{1}{1+I}\)
- Complementary sensitivity transfer function \(\frac{I}{1+I}\)

Internal model principle
- \(I_{II}\) should have \(-20\text{dB/decade}\) slope in the DC region for being Type I
- \(-20\text{dB/decade}\) slope in the DC region for Type II
- \(-60\text{dB/decade}\) slope in the DC region for Type III

\[
P_c = \lim_{s \to 0} |L(s)|
\]

\[
P_v = \lim_{s \to 0} |sL(s)| ; \quad K_a = \lim_{s \to 0} |s^2L(s)|
\]

\[
\text{Step Tracking error} = \frac{1}{1+K_p}
\]

\[
\text{overshoot} \quad \left(\frac{1}{k_v}ight)
\]

\[
\text{settling time} \quad \left(\sqrt{k_a}\right)
\]

Time domain specifications
These are specified with respect to the step response of the system.
Look at a prototype second order example.

Second order Prototype

\[ \frac{1}{\frac{s^2}{s^2 + 2\xi \omega_0 s + \omega_0^2}} \]

\[ G(s) = \frac{\omega_0^2}{s(s + 2\xi \omega_0)} \quad 0 < \xi < 1 \]

- The closed-loop map from \( r \rightarrow y \) is

\[ \frac{\omega_0^2}{s^2 + 2\xi \omega_0 s + \omega_0^2} \]

Step response of the closed-loop system has the following form

\[ y(t) = 1 \left[ 1 - e^{-\xi \omega_0 t} \sin(\omega_d t + \phi) \right] \]

where \( \omega_d = \omega_0 \sqrt{1 - \xi^2} \) - damped natural frequency

\[ \cos \phi = \xi \]

\[ \frac{y(t)_{TP} - 1}{1} \]

TR = rise time, the first time the output hits the value 1

\[ TR = \frac{\pi}{\omega_0 \sqrt{1 - \xi^2}} \]

- \( t_{\text{tp}} \) is the time when the maximum is reached.
Open-loop response and its connection to closed-loop behavior.

Frequency response:

Closed-loop gain:

What is desired is that $t_b$ to be small.

$\text{settling time is the time when the maximum is reached}$.

$\text{response time}$.
Wgc can be obtained by solving

\[ g_c(j\omega_c) = 1 \]

This yields

\[ \omega_c = \omega_0 \sqrt{1 + 4q^4 - 2q^2}^{1/2} \]

1. \( q = 0 \); \( \omega_c = \omega_0 \)

2. \( q = 1 \); \( \omega_c = \sqrt{0.24} \omega_0 \)

3. \( q = 0.907 \); \( \omega_c = \sqrt{0.41} \omega_0 \)

4. \( q = 0.5 \); \( \omega_c = \sqrt{0.62} \omega_0 \)

A good rule of thumb is that

\[ \omega_0 = 1.4 \omega_c \approx (1.2 - 1.4) \omega_c \]

\[ G_u = \frac{\omega_0^2}{s^2 + 2q\omega_0s + \omega_0^2} \]; the 3dB point is obtained by

\[ |G_u(j\omega)|^2 = \frac{1}{2} \]

This yields

\[ \omega_{BW} = \omega_0 \sqrt{1 - 2q + \sqrt{2 + 4q^2}} \]

\[ \omega_{BW} \approx (1.2 - 1.6) \omega_c \]

Gain crossover of \( L \).

Bandwidth of the closed-loop system

Even though the relation of \( \omega_c \) (open-loop characteristic) and \( \omega_{BW} \) (closed-loop characteristic) held exactly for 2nd order prototype, this relation guides the intuition for a general \( L \) that is not necessarily 2nd order prototype.
Phase Margin for the second order prototype:

\[ PM = \frac{T_1}{2} - \arctan \left( \frac{\sqrt{1+4\zeta^2} - 2\zeta^2}{2\zeta} \right) \]

\[ \zeta = 0.5 \quad PM = 51^\circ \]
\[ \zeta = 0.6 \quad PM = 59^\circ \]
\[ \zeta = 0.7 \quad PM = 65^\circ \]
\[ \zeta = 1 \quad PM = 76^\circ \]

A good rule of thumb is

\[ PM = 100\zeta \quad \text{in degrees} \]

- The maximum fraction overshoot

\[ MP = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \]

depends only on \( \zeta \).

If \( MP \) is specified then essentially only \( \zeta \) and therefore on \( PM \).

This relationships are exact for a second order system. Nevertheless they guide our intuition for a general system.

Let's assume an open loop bode plot (not necessarily a second order)
**Closed loop Specs**

1. **Bandwidth** $w_{bnl}$
2. **PM (in degrees)** $\approx 100\,$°
3. **GM**
4. **$\mu_p = e^{-\pi^2(1-\eta^2)}$**
5. **$\tau_s = \frac{4}{\omega_0}$**
6. **$\tau_r = \frac{\pi - \omega_0^2}{\omega_0(\sqrt{1-\eta^2})}$**
7. **Steady state behavior**

**Open loop**

\[(1.2 \times 1.6) \omega_{gc} = \omega_{bnl}\]

Can be read from plot curve.

Can be read from the Bode plot.

\[\eta = \frac{PM\text{ index}}{100}\]

\[\omega_0 = 1.4 \omega_{gc}\]

\[\omega_{bc} = 1.4 \omega_{gc}\]

\[\gamma = \frac{PM\text{ index}}{100}\]

determined by the behavior of $L(s)$ near $\omega = 0$.

**Controller design**

$Specs$ on closed loop $\rightarrow$ $Specs$ on $L$ (loop-shape of $L$)
Design Controller $K$ such that $\mathcal{L}K = 1$

**Proportional Controller**

Suppose the Spec is $M_p \leq 0.16$.

First translate $M_p \leq 0.16 \Rightarrow \lim_{s \to s_0} G(s) \leq 0.16$.

A PM $> PM_a$.

Find $\omega_{gcd}$ such that $L(\omega_{gcd}) + \pi = PM_a$.

And choose $K$ such that

$\left| K G(\omega_{gcd}) \right| = 1$

**Proportional Integral Controller**

$K = K_p + \frac{K_i}{s} = \frac{K_i}{s} \left[ 1 + \frac{s}{K_i/K_p} \right]$. 

$20\log K_p$