HW 4

October 13, 2010

1. Find the Laplace transforms of the following functions. Use the theorems on Laplace transforms, if applicable.

   (a) $g(t) = 5te^{-5t}u_s(t)$

   (b) $g(t) = (t \sin 2t + e^{-2t})u_s(t)$

   (c) $g(t) = 2e^{-2t} \sin 2t u_s(t)$

   (d) $g(t) = \sin 2t \cos 2t u_s(t)$

   (e) $g(t) = \sum_{k=0}^{\infty} e^{-5kT} \delta(t - kT)$ where $\delta(t)$ = unit-impulse function.

2. Find the inverse Laplace transforms of the following functions. First, perform partial-fraction expansion on $G(s)$; then use the Laplace transform table.

   (a) $G(s) = \dfrac{1}{s(s+2)(s+3)}$

   (b) $G(s) = \dfrac{10}{(s+1)^2(s+3)}$

   (c) $G(s) = \dfrac{100(s+2)}{s(s^2+4)(s+1)}e^{-s}$

   (d) $G(s) = \dfrac{2(s+1)}{s(s^2+s+2)}$

   (e) $G(s) = \dfrac{1}{(s+1)^3}$

   (f) $G(s) = \dfrac{2(s^2+s+1)}{s(s+1.5)(s^2+5s+5)}$

   (g) $G(s) = \dfrac{2+2se^{-s}+4e^{-2s}}{s^2+3s+2}$

3. The following differential equations represent linear time-invariant systems, where $r(t)$ denotes the input and $y(t)$ the output. Find the trasnfer function $Y(s)/R(s)$ for each of the systems. (Assume zero initial conditions.)

(a) $\dfrac{d^3y(t)}{dt^3} + 2\dfrac{d^2y(t)}{dt^2} + 5\dfrac{dy(t)}{dt} + 6y(t) = 3\dfrac{dr(t)}{dt} + r(t)$

(b) $\dfrac{d^4y(t)}{dt^4} + 10\dfrac{d^2y(t)}{dt^2} + \dfrac{dy(t)}{dt} + 5y(t) = 5r(t)$

(c) $\dfrac{d^3y(t)}{dt^3} + 10\dfrac{d^2y(t)}{dt^2} + 2\dfrac{dy(t)}{dt} + y(t) + 2\int_0^t y(\tau)d\tau = \dfrac{dr(t)}{dt} + 2r(t)$

(d) $2\dfrac{d^2y(t)}{dt^2} + \dfrac{dy(t)}{dt} + 5y(t) = r(t) + 2r(t-1)$

(e) $\dfrac{d^2y(t+1)}{dt^2} + 4\dfrac{dy(t+1)}{dt} + 5y(t+1) = \dfrac{dr(t)}{dt} + 2r(t) + 2\int_{-\infty}^t r(\tau)d\tau$

(f) $\dfrac{d^3y(t)}{dt^3} + 2\dfrac{d^2y(t)}{dt^2} + \dfrac{dy(t)}{dt} + 2y(t) + 2\int_{-\infty}^t y(\tau)d\tau = 3\dfrac{dr(t-2)}{dt} + 2r(t-2)$