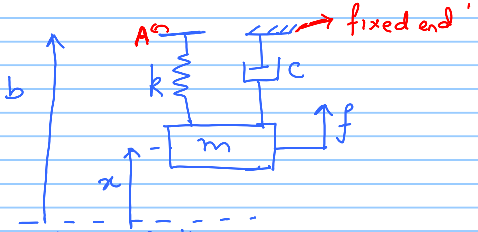
Homework 1:

Note Title 9/16/2009

Problem 1:

Consider the Spring-mass-damper System shown below



The position of the mass m is given by x

One end of the damper is connected to a fixed end and another to the mass m.

One end of the Spring is connected to a movable base A whose position is given by b.

There is an external force f acting on the mass m

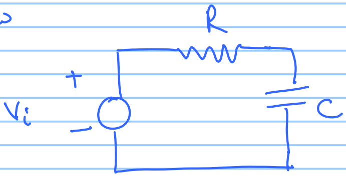
(a) Derive the equation of motion of the mass m.

(b) Derive a State-Space representation of the dynamics

(c) With $m = 1\,kg$; $k = 1\,N/m$; $C = 0.1$ use MATLAB to obtain the response of the System when b is a unit step; $f = 0$ and initial conditions is 0.

Problem 2:

Consider the RLC Circuit given below



(i) Derive the differential equation that governs the dynamics of the voltage across the Capacitor V_c.

(ii) Obtain the state space description of the system with the state being $\begin{bmatrix} V_c \\ \frac{dV_c}{dt} \end{bmatrix}$

(iii) Obtain the response of the voltage across the resistor when V_i is a unit step: $V_i(t) = 1$ if $t \geq 0$
$= 0$ if $t < 0$

with zero initial conditions. Plot the response.

Problem 3:

Consider the following system that shows a mass m with position x.

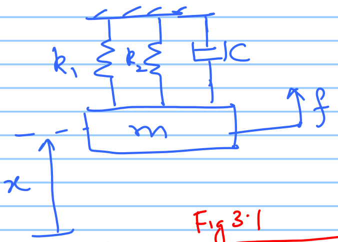


Fig 3·1

(a) Derive the equation of motion of the mass m.

(b) Consider the system

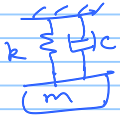
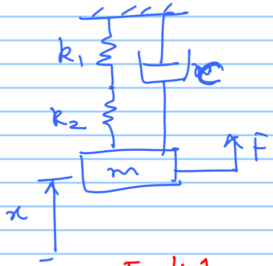


Fig 3·2

For what value of k is the system in Figure 3·1 equivalent to the system in Fig. 3·2.

Problem 4.



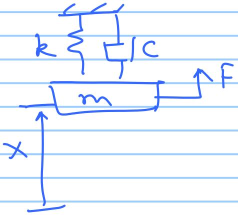


Fig 4.1

(a) Obtain the equation of motion in Figure 4.1

(b) For what value of k is system in Figure 4.2 equivalent to the system in Fig. 4.1

Problem 5:

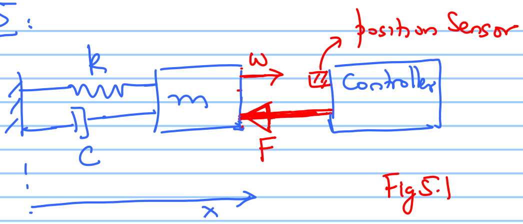


Fig 5.1

The position x of the mass 'm' is sensed by a position sensor and the controller exerts a force opposing the motion of the mass m with a magnitude $k_c x$. The mass m is also subject to an external force w as shown.

(a) Obtain the equation of motion of mass m

(b) What is the value of the stiffness k of the system below such that the system in Figure 5.1 is equivalent to the system in Figure 5.2