EE 5505 Prof. Jindal
Wireless Communication March 2, 2010

Homework 5
Due: Tuesday, March 9, 11:15 AM

Please turn in your MATLAB scripts in addition to your solutions and plots.

1. In this problem we will investigate performance with simple ARQ (i.e., packets are retransmitted when they are decoded incorrectly), and particularly will focus on finding the transmission rate/packet error probability that maximizes the goodput when ARQ is used. For the sake of simplicity we will focus on a scenario where there is no time or frequency diversity during transmission of a codeword (i.e., an ARQ round).

   (a) The outage probability (i.e., the probability a packet is decoded correctly) for transmitted rate R (with no diversity) is given by:

   $$P_{out}(R) \quad = \quad \mathbb{P}\left[\log_2(1+|h|^2\text{SNR}) < R\right]$$

   Assuming Rayleigh fading, show that the transmitted rate that achieves an outage probability equal to ϵ is:

   $$R_\epsilon \quad = \quad \log_2\left(1+\log_e\left(\frac{1}{1-\epsilon}\right)\text{SNR}\right). \tag{1}$$

   (b) In class, we found that the long-term average (successful) rate with simple ARQ is the product of the transmitted rate and the packet success probability, i.e., $R_\epsilon(1-\epsilon)$. Define $R^\star$ as the transmitted rate that maximizes the long-term average rate:

   $$R^\star = \text{argmax}_\epsilon \quad R_\epsilon(1-\epsilon). \tag{2}$$

   for a particular value of SNR. Show that

   $$R^\star = \log_2(e)W(\text{SNR}) \tag{3}$$

   where $W(\cdot)$ is the Lambert function (a definition of the Lambert function can be found on the web). Plot $R^\star$ and the corresponding outage probability versus SNR (in dB) for 0 to 20 dB.

   (c) We will now consider the same rate optimization assuming that the receiver has two antennas and MRC is performed. In this case the outage probability expression becomes:

   $$P_{out}(R) \quad = \quad \mathbb{P}\left[\log_2(1+\|\mathbf{h}\|^2\text{SNR}) < R\right]$$

   where $\mathbf{h}$ is a 2-dimensional vector with iid complex normal, unit variance components. Find an explicit expression for the long-term average rate in terms of the transmitted rate R.

(d) Using the expression derived in the previous part, numerically compute (using Matlab) the optimal transmitted rate $R^\star$ (and the corresponding outage probability) for SNR ranging from 0 to 20 dB. Plot the optimal outage probability with no antenna diversity (from part b) and the optimal outage probability with two antennas, versus SNR. Compare the two and explain the difference.

(e) Infer how the optimal outage probability changes as the number of receive antennas is increased beyond 2.

2. In this problem we will investigate the optimal waterfilling policy (that maximizes the rate assuming perfect CSIT and that the transmitter is subject to an average power constraint) for a channel in which the fading power (i.e., $|h|^2$) is equal to b with probability 1/2 and is equal to g with probability 1/2, with $b < g$ (b/g represent bad/good).

   (a) Find a closed form expression for the power allocated to states b and g, in terms of SNR, as well as an expression for the corresponding rates in each of the two states.

   (b) Plot the rate achieved with the optimal policy and with equal power, versus SNR, for $b = 0.1$ and $g = 1.9$. Also include the AWGN capacity in the plot, and comment on the ordering between the three curves.

   (c) Prove that the rate achieved with the optimal power policy converges (absolutely) to the rate achieved with equal power allocation as SNR $\to \infty$. In other words, show that the difference between the rate achieved with the optimal power policy and with equal power goes to zero as SNR $\to \infty$.

3. Write a Matlab program that uses Monte Carlo simulation to estimate the probability of bit error of QPSK in Rayleigh fading, assuming that the receiver treats the noisy observation of the channel fade h:

$$h + \frac{1}{\sqrt{\beta \text{SNR}}} z$$

   as if it were the actual channel coefficient, where z is unit variance complex Gaussian. In other words, the receiver adjusts the phase according to the noisy observation of h (instead of according to the true h), and then uses the standard decision regions of QPSK. Produce a plot of bit error probability versus β for β ranging from 1 to 20, for SNR = 0, 5, and 10 dB. Comment on how quickly the bit error probability approaches the bit error probability with perfect channel knowledge.