EE 5505 Prof. N. Jindal
Wireless Communication April 13, 2010

Homework 8
Due: Tuesday, April 20, 11:15 AM

1. You are provided Matlab code that computes the ergodic capacity of a MIMO channel without CSIT and with CSIT (and SVD-based waterfilling), and produces plots of capacity versus SNR.

   (a) Modify the code so that it also plots the rate achieved if the TX has CSIT and allocates all of its available power to the eigenmode (i.e., spatial channel) with the largest eigenvalue.

   (b) Using the approximation $\log(1+x) \approx x$, argue that the optimal waterfilling strategy at low SNR is to allocate all power to the strongest eigenmode.

   (c) For $n_T = n_R = 4$, plot the ergodic capacity with and without CSIT along with the rate for beamforming in the strongest eigenmode. Explain the differences between the curves.

   (d) Again using the approximation $\log(1+x) \approx x$, argue that the ergodic capacity with no CSIT for an $N \times N$ system is about the same as that of a $1 \times N$ system at low SNR.

   (e) Add curves for $n_T = 1$, $n_R = 4$, and for $n_T = 4$, $n_R = 1$. Indicate which curves are identical; for curves that are not identical, explain why they differ.

2. This problem is intended to illustrate the high complexity of performing joint decoding across the transmitted signals. Consider an $N \times N$ system in which independent BPSK signals ($\pm\sqrt{E_s/N}$) are transmitted from each of the N transmit antennas. The received vector $\mathbf{y}$ follows the standard model:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{z}$$

   where $\mathbf{z}$ has independent complex normal components (variance N_0).

   Find an expression for the log-likelihood ratio of the bit sent on the i-th transmit antenna:

$$\log\left(\frac{\mathbb{P}[x_i = +\sqrt{E_s/N}|\mathbf{y}]}{\mathbb{P}[x_i = -\sqrt{E_s/N}|\mathbf{y}]}\right).$$

   (The log-likelihood ratio is the sufficient statistic to be passed to the channel decoder).

3. In our discussion of BLAST architectures, in which separate data symbols are sent from each transmit antenna, we have always assumed that separate codewords are transmitted from each transmit antenna. An alternative is to generate a single codeword, and then to transmit part of the codeword from the first antenna, a second part from the second antenna, and so forth (this is referred to as coding across streams).

   (a) Explain why MMSE-SIC would not work very well if coding is done across the streams.

   (b) Describe how MMSE without SIC could be used with coding across streams.