EE 8950 Prof. N. Jindal
Advanced Topics in Wireless Communication Feb. 13, 2008

Homework 2

Due: Tuesday, Feb. 19, 5:00 PM

1. In this problem we will derive expressions for the (ergodic) mutual information achieved by a finite input constellation in a fading channel with perfect CSIR. Recall that our standard fading channel is written as:

$$y_k = h_k x_k + z_k$$

where the fading coefficient h_k and the input x_k are assumed to be independent. Perfect CSIR is modeled by treating the pair (y_k, h_k) as the output at time k.

(a) The basic definition of mutual information gives:

$$I(x_k; y_k, h_k) = E_{X,Y,H}\left[\log\left(\frac{p(x,y,h)}{p(x)p(y,h)}\right)\right].$$

Show that this expression can be simplified to:

$$I(x_k; y_k, h_k) = E_{X,Y,H}\left[\log\left(\frac{p(y|x,h)}{p(y|h)}\right)\right].$$

(b) If the input is 4-QAM, it is sufficient to compute the mutual information achieved per dimension. Show that for 2-PAM, the mutual information per dimension can be written as:

$$I(x_k; y_k, h_k) = E_{X,Y,H}\left[\log\left(\frac{2\exp\left(\frac{1}{2}(y-|h|x)^2\right)}{\exp\left(\frac{1}{2}(y-|h|\sqrt{SNR})^2\right)+\exp\left(\frac{1}{2}(y+|h|\sqrt{SNR})^2\right)}\right)\right]$$

(c) Use the above expression to numerically compute the mutual information achieved by 4-QAM in Rayleigh fading, and plot this quantity vs. SNR. (The easiest way to do this is by Monte Carlo simulation: draw (x, y, h, z) by drawing x, h, and z independently according to their respective distributions, and then using $y = hx + z$) In your plot also include the ergodic capacity, which for Rayleigh fading can be written in closed form as:

$$C = E[\log_2(1 + SNR|h|^2)] = \log_2(e)e^{1/SNR}E_1\left(\frac{1}{SNR}\right)$$

where $E_1(\cdot)$ is the exponential integral function, which is defined by $E_1(x) = \int_x^\infty \frac{e^{-t}}{t}dt$ and can be evaluated in MATLAB using the EXPINT command.

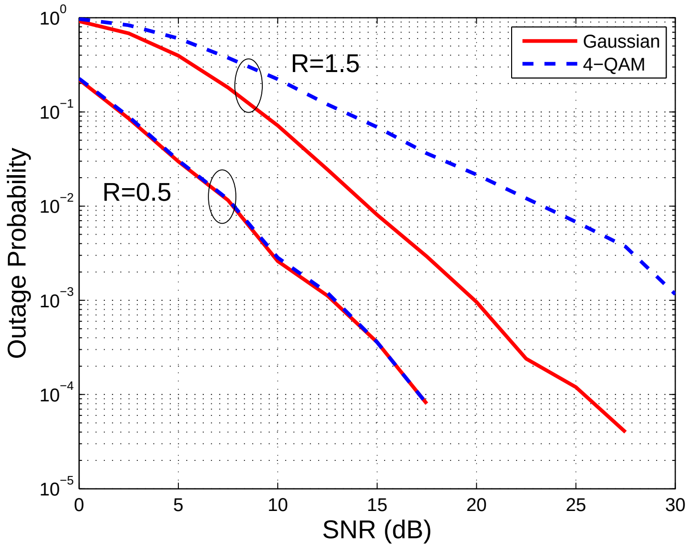

Figure 1: Outage Prbability vs. SNR.

2. In class we saw that an appropriately chosen QAM constellation can achieve a mutual information (averaged over the fading distribution) quite close to the ergodic capacity of a Rayleigh-faded channel. In this question we will see that the same is generally true for the outage capacity of block fading channels, although we need to be more careful regarding constellation size.

   For a Gaussian input, the mutual information for block fading with diversity order L is:

$$P_{\text{out}}(R) = P\left[\frac{1}{L}\sum_{i=1}^{L}\log_2(1 + SNR|h_i|^2) < R\right]$$

   while for a finite input the outage probability is given by:

$$P_{\text{out}}(R) = P\left[\frac{1}{L}\sum_{i=1}^{L}I(X;Y|h = h_i) < R\right]$$

   where $I(X;Y|h = h_1)$ is the mutual information achieved by the input when the fading coefficient is equal to h_1 (the quantity $I(X;Y|h = h_1)$ is a deterministic function of h_1). In both expressions the probability is computed with respect to $h_1, \ldots, h_L$, which are iid and unit-variance complex normals.

   In Fig. 1, outage probability is plotted versus SNR for a Gaussian input and a 4-QAM input for $L = 2$ and two different rates: $R = \frac{1}{2}$ and $R = \frac{3}{2}$.

   (a) Given an intuitive explanation why the outage probability of the Gaussian input decreases with SNR as SNR^{-2}.

   (b) Explain why the outage probability for 4-QAM decreases with SNR as SNR^{-2} for $R = \frac{1}{2}$, but decreases only as SNR^{-1} for $R = \frac{3}{2}$.