

# Bandwidth-SINR Tradeoffs in Spatial Networks

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**Abstract**—This paper addresses the following question, which is of interest in the design of a multiuser decentralized network: given a total system bandwidth of  $W$  Hz and a fixed data rate constraint of  $R$  bps for each transmission, how many frequency slots  $N$  of size  $W/N$  should the band be partitioned into to maximize the number of simultaneous transmissions in the network? Dividing the available spectrum reduces the number of users on each band and therefore decreases multi-user interference level, but also increases the SINR requirement for each transmission because the same information rate must be achieved over a smaller bandwidth. Exploring this tradeoff between bandwidth and SINR and determining the optimum value of  $N$  in terms of the system parameters is the focus of the paper. Using stochastic geometry, we analytically derive the optimal SINR threshold on this tradeoff curve and show that it is a function of only the path loss exponent. Furthermore, the optimal SINR point lies between the low-SINR (power-limited) and high-SINR (bandwidth-limited) regimes.

## I. INTRODUCTION

We consider a spatially distributed network, representing either a wireless ad hoc network or unlicensed (and uncoordinated) spectrum usage by many nodes (e.g., WiFi), and consider the tradeoff between bandwidth and SINR. We ask the following question: given a fixed total system bandwidth and a fixed rate requirement for each single-hop transmitter-receiver link in the network, at what point along the bandwidth-SINR tradeoff-curve should the system operate at in order to maximize the spatial density of transmissions? For example, given a system-wide bandwidth of 1 Hz and a desired rate of 1 bit/sec, should (a) each transmitter utilize the entire spectrum (e.g., transmit one symbol per second) and thus require an SINR of 1 (utilizing  $R = W \log(1 + SINR)$  if interference is treated as noise), (b) the band be split into two orthogonal 0.5 Hz sub-bands where each transmitter utilizes one of the sub-bands with the required SINR equal to 3, or (c) the band be split into  $N > 2$  orthogonal  $\frac{1}{N}$  Hz sub-bands where each transmitter utilizes one of the sub-bands with the required SINR equal to  $2^N - 1$ ?

We consider a network with the following key characteristics:

- Transmitter node locations are a realization of a homogeneous spatial Poisson process.
- Each transmitter communicates with a single receiver that is a reference distance  $d$  meters away.
- All transmissions are constrained to have an absolute rate of  $R$  bits/sec regardless of the bandwidth.
- All multi-user interference is treated as noise.

- The channel is frequency-flat, reflects path-loss and possibly fast and/or slow fading, and is constant for the duration of a transmission.
- Transmitters do not have channel state information and no transmission scheduling is performed, i.e., transmissions are independent and random (e.g., ALOHA)

The last assumption should make it clear that we are considering only an *off-line* optimization of the frequency band structure, and that no on-line (e.g., channel- and queue-based) transmission or sub-band decisions are considered.

## A. Related Work

The transmission capacity framework introduced in [1] is used to quantify the throughput of such a network, since this metric captures notions of spatial density, data rate, and outage probability, and is more amenable to analysis than the more popular transport capacity [2]. Using tools from stochastic geometry, the distribution of interference from other concurrent transmissions at a reference receiving node<sup>1</sup> is characterized as a function of the spatial density of transmitters, the path-loss exponent, and possibly the fading distribution. The distribution of SINR at the receiving node can then be computed, and an outage occurs whenever the SINR falls below some threshold  $\beta$ . The outage probability is clearly an increasing function of the density of transmissions, and the transmission capacity is defined to be the maximum density of successful transmissions such that the outage probability is no larger than some prescribed constant  $\epsilon$ .

The problem studied in this work is essentially the optimization of spatial frequency reuse in uncoordinated (ad hoc) networks, which is a well studied problem in the context of cellular networks (see for example [3] and references therein). A key difference is that planned frequency reuse patterns can be used in cellular networks while this is not possible in an ad hoc network. There has been prior work on frequency reuse in ad-hoc networks, e.g., [4], but this appears to be the first analytical derivation of optimal reuse. The issue of optimal reuse for ad hoc networks is considered in [5] for infinitely dense networks, but this scenario differs drastically from the finite density network we consider here.

<sup>1</sup>The randomness in interference is only due to the random positions of the interfering nodes and fading.

## II. KEY INSIGHTS

The bandwidth-SINR tradeoff reveals itself if the system bandwidth is split into  $N$  non-overlapping bands and each transmitter transmits on a randomly chosen band with some fixed power (independent of  $N$ ). This splitting of the spectrum results in two competing effects. First, the density of transmitters on each band is a factor of  $N$  smaller than the overall density of transmitters, which reduces interference and thus increases SINR. Second, the threshold SINR must be increased in order to maintain a fixed rate while transmitting over  $\frac{1}{N}$ -th of the bandwidth. Although intuition from point-to-point AWGN channels might indicate that the optimum solution is to not split the band ( $N = 1$ ), this is generally quite far from the optimum. Our analysis shows that  $N$  should be chosen such that the required threshold SINR lies between low-SNR (power-limited) and high-SNR (bandwidth-limited).

The intuition behind this result is actually quite simple: if  $N$  is such that the threshold SINR is in the wideband regime (roughly below 0 dB), then doubling  $N$  leads to an approximate doubling (in linear units) of the threshold SINR. If the path-loss exponent is strictly greater than 2, doubling the threshold SINR reduces the allowable intensity of transmissions on each band by a factor strictly smaller than two. However, the total intensity is exactly twice the per sub-band density. The combination of these effects is a net increase in the allowable intensity of transmissions, and therefore it is beneficial to increase  $N$  until the required SINR threshold begins to increase *exponentially* rather than *linearly* with  $N$ .

## III. PRELIMINARIES

### A. System Model

We consider a set of transmitting nodes at an arbitrary snapshot in time with locations specified by a homogeneous Poisson process of intensity  $\lambda$  on the infinite two-dimensional plane. We consider a reference receiver that is located, without loss of generality, at the origin, and let  $X_i$  denote the distance of the  $i$ -th transmitting node to the reference receiver. The reference transmitter is placed a fixed distance  $d$  away. Received power is modeled by path loss with exponent  $\alpha > 2$  and a distance-independent fading coefficient  $h_i$  (from the  $i$ -th transmitter to the reference receiver). Therefore, the SINR at the reference receiver is:

$$SINR_0 = \frac{\rho d^{-\alpha} |h_0|}{\eta + \sum_{i \in \Pi(\lambda)} \rho X_i^{-\alpha} |h_i|},$$

where  $\Pi(\lambda)$  indicates the point process describing the (random) interferer locations, and  $\eta$  is the noise power. If Gaussian signaling is used, the mutual information conditioned on the transmitter locations and fading realizations is:

$$I(X_0; Y_0 | \Pi(\lambda), \mathbf{h}) = \log_2(1 + SINR_0),$$

where  $\mathbf{h} = (h_0, h_1, \dots)$ . Notice that we assume that all nodes simultaneously transmit with the same power  $\rho$ , i.e., power control is not used. Moreover, nodes decide to transmit independently and irrespective of their channel conditions, which corresponds roughly to slotted ALOHA.

### B. Transmission Capacity Model

In the outage-based transmission capacity framework, an outage occurs whenever the SINR falls below a prescribed threshold  $\beta$ , or equivalently whenever the instantaneous mutual information falls below  $\log_2(1 + \beta)$ . Therefore, the system-wide outage probability is:

$$P \left( \frac{\rho d^{-\alpha} |h_0|}{\eta + \sum_{i \in \Pi(\lambda)} \rho X_i^{-\alpha} |h_i|} \leq \beta \right).$$

This quantity is computed over the distribution of transmitter positions as well as the iid fading coefficients, and thus corresponds to fading that occurs on a slower time-scale than packet transmission. The outage probability is clearly an increasing function of the intensity  $\lambda$ .

If  $\lambda(\epsilon)$  is the maximum intensity of *attempted* transmissions such that the outage probability (for a fixed  $\beta$ ) is no larger than  $\epsilon$ , then the transmission capacity is then defined as  $c(\epsilon) = \lambda(\epsilon)(1 - \epsilon)b$ , which is the maximum density of *successful* transmissions times the spectral efficiency  $b$  of each transmission. In other words, transmission capacity is like area spectral efficiency subject to an outage constraint. Using tools from stochastic geometry, in [1] it is shown that the maximum spatial intensity  $\lambda(\epsilon)$  for small values of  $\epsilon$  is:

$$\lambda(\epsilon) = \frac{c}{\pi d^2} \left( \frac{1}{\beta} - \frac{\eta}{\rho d^{-\alpha}} \right)^{\frac{2}{\alpha}} \epsilon + O(\epsilon^2), \quad (1)$$

where  $c$  is a constant that depends only on the fading distribution [6]. Because fading has only a multiplicative effect, it does not effect the SINR-bandwidth tradeoff and thus is not considered in the remainder of the paper.

## IV. OPTIMIZING FREQUENCY USAGE

In this section we consider a network with a fixed total bandwidth of  $W$  Hz, and where each link has a rate requirement of  $R$  bits/sec and an outage constraint  $\epsilon$ . Assuming the network operates as described in the previous section, the goal is to determine the number of sub-bands  $N$  for which the maximum density of transmissions can be supported.

### A. Definitions and Setup

In performing this analysis, we assume that there exist coding schemes that operate at any point along the AWGN capacity curve.<sup>2</sup> We define the *spectral utilization*  $\tilde{R}$  as the ratio between the required rate and total bandwidth:

$$\tilde{R} \triangleq \frac{R}{W} \text{ bps/Hz/user.}$$

We intentionally refer to  $\tilde{R}$ , which is externally defined, as the spectral utilization; the *spectral efficiency*, on the other hand, is a system design parameter determined by the choice of  $N$ .

If the system bandwidth is not split ( $N = 1$ ), each node utilizes the entire bandwidth of  $W$  Hz. Therefore, the required

<sup>2</sup>It is straightforward to show that relaxing this assumption by allowing for operation at a constant coding gap from AWGN capacity has no effect on our analysis.

SINR  $\beta$  is determined by inverting the standard rate expression:  $R = W \log_2(1 + \beta)$ , which gives  $\beta = 2^{\frac{R}{W}} - 1 = 2^{\tilde{R}} - 1$ . The maximum intensity of transmissions can be determined by plugging in this value of  $\beta$  into (1), along with the other relevant constants.

If the system bandwidth is split into  $N$  orthogonal sub-bands each of width  $\frac{W}{N}$ , and each transmitter-receiver pair uses only one of these sub-bands at random, the required SINR  $\beta(N)$  is determined by inverting the rate expression  $R = \frac{W}{N} \log_2(1 + \beta(N))$  which yields:

$$\beta(N) = 2^{\frac{N\tilde{R}}{W}} - 1 = 2^{N\tilde{R}} - 1.$$

Notice that the spectral efficiency (on each sub-band) is  $b = \frac{R}{W/N}$  bps/Hz, which is  $N$  times the spectral utilization  $\tilde{R}$ . The maximum intensity of transmissions *per sub-band* for a particular value of  $N$  is determined by plugging  $\beta(N)$  into (1) with noise power  $\eta = \frac{W}{N}N_0$ . Since the  $N$  sub-bands are statistically identical, the maximum total intensity of transmissions, denoted by  $\lambda(\epsilon, N)$ , is the per sub-band intensity multiplied by a factor of  $N$ . Dropping the second order term in (1), we have:

$$\lambda(\epsilon, N) \approx N \left( \frac{\epsilon}{\pi d^2} \right) \left( \frac{1}{\beta(N)} - \frac{1}{N \cdot SNR} \right)^{\frac{2}{\alpha}}, \quad (2)$$

where the constant  $SNR \triangleq \frac{\rho d^{-\alpha}}{N_0 W}$  is the signal-to-noise ratio in the absence of interference when the entire band is used.

### B. Optimization

Optimizing the number of sub-bands  $N$  therefore reduces to the following one-dimensional maximization:

$$N^* = \arg \max_N \lambda(\epsilon, N), \quad (3)$$

which yields a solution that depends only on the path-loss exponent  $\alpha$ , the spectral utilization  $\tilde{R}$ , and the constant  $SNR$ .

In general, the interference-free  $SNR$  can be ignored because the systems of interest are interference- rather than noise-limited. Assuming  $SNR$  is infinite we have:

$$\lambda(\epsilon, N) \approx \left( \frac{\epsilon}{\pi d^2} \right) N \cdot \beta(N)^{-\frac{2}{\alpha}} \quad (4)$$

$$= \left( \frac{\epsilon}{\pi d^2} \right) N (2^{N\tilde{R}} - 1)^{-\frac{2}{\alpha}}. \quad (5)$$

Since  $\tilde{R}$  is a constant, we make the substitution  $b = N\tilde{R}$  and equivalently solve:

$$\max_{b>0} b(2^b - 1)^{-\frac{2}{\alpha}}. \quad (6)$$

By taking the derivative and solving appropriately, it is straightforward to show the optimal  $b^*$  satisfies:

$$b^* = (\log_2 e) \frac{\alpha}{2} (1 - e^{-b^*}), \quad (7)$$

which has solution

$$b^* = \log_2 e \left[ \frac{\alpha}{2} + W \left( -\frac{\alpha}{2} e^{-\frac{\alpha}{2}} \right) \right], \quad (8)$$

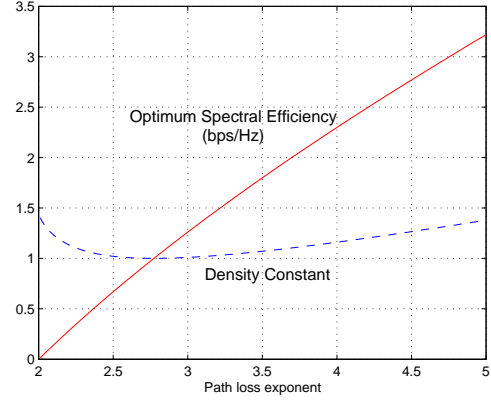


Fig. 1. Optimal Spectral Efficiency vs. Path-Loss Exponent

where  $W(z)$  is the principle branch of the Lambert  $W$  function and thus solves  $W(z)e^{W(z)} = z$ .<sup>3</sup> It is easily shown that  $b^*$  is an increasing function of  $\alpha$ , is upper bounded by  $\frac{\alpha}{2} \log_2 e$ , and that  $b^*/(\frac{\alpha}{2} \log_2 e)$  converges to 1 as  $\alpha$  grows large.

Recalling that  $b = N\tilde{R}$  is the spectral efficiency on each sub-band, the quantity  $b^*$ , which is a function of only the path-loss exponent  $\alpha$ , is the *optimum spectral efficiency*.<sup>4</sup> Therefore, the optimal value of  $N$  (ignoring the integer constraint) is determined by simply dividing the optimal spectrum efficiency  $b^*$  by the spectral utilization  $\tilde{R}$ :

$$N^* = \frac{b^*}{\tilde{R}}. \quad (9)$$

To take care of the integer constraint on  $N$ , the nature of the derivative of  $b(2^b - 1)^{-\frac{2}{\alpha}}$  makes it sufficient to consider only the integer floor and ceiling of  $N^*$  in (9). If the spectral utilization is larger than the optimum spectral efficiency, i.e.,  $\tilde{R} \geq b^*$ , then choosing  $N = 1$  is optimal. On the other hand, if  $\tilde{R} \leq \frac{1}{2}b^*$ , then the optimal  $N$  is strictly larger than 1. In the intermediate regime where  $\frac{1}{2}b^* \leq \tilde{R} \leq b^*$ , the optimal  $N$  is either one or two.

In Fig. 1 the optimal spectral efficiency  $b^*$  is plotted (in units of bps/Hz) as a function of the path-loss exponent  $\alpha$ , along with the quantity  $b^*(2^{b^*} - 1)^{-\frac{2}{\alpha}}$ , which is referred to as the density constant because the optimal density  $\lambda^*(\epsilon)$  is this quantity multiplied by  $\left( \frac{\epsilon}{R\pi d^2} \right)$ . The optimal spectral efficiency is very small for  $\alpha$  close to 2 but then increases nearly linearly with  $\alpha$ ; for example, the optimal spectral efficiency for  $\alpha = 3$  is 1.26 bps/Hz (corresponding to  $\beta = 1.45$  dB).

<sup>3</sup>Equation (8) is nearly identical, save for a factor of 2, to the expression for the optimal number of hops in an interference-free linear network given in equation (18) of [4]. This similarity is due to the fact that the objective function in equation (17) of [4] coincides almost exactly with (5).

<sup>4</sup>An optimal spectral efficiency is derived for interference-free, regularly spaced, 1-D networks in [7]; however, these results differ by approximately a factor of 2 from our results due to the difference in the network dimensionality.

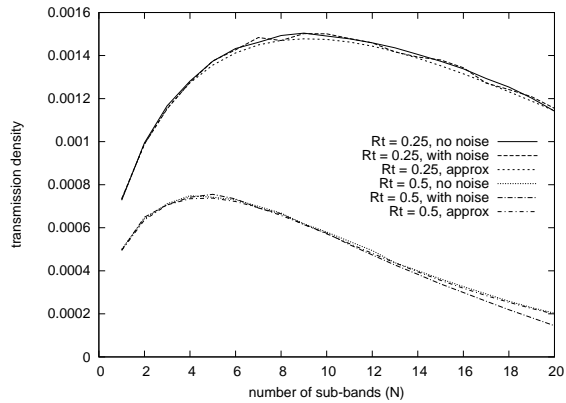


Fig. 2. Optimal Spectral Efficiency vs. Path-Loss Exponent

### C. Interpretation

To gain an intuitive understanding of the optimal solution, first consider the behavior of  $\lambda(\epsilon, N)$  when the quantity  $N\tilde{R}$  is small, i.e.  $N\tilde{R} \ll 1$ . In this regime, the SINR threshold  $\beta(N)$  grows approximately linearly with  $N$ :

$$\begin{aligned} \beta(N) = 2^{N\tilde{R}} - 1 &= e^{N\tilde{R}\log_e 2} - 1 \\ &\approx N\tilde{R}\log_e 2. \end{aligned}$$

Plugging into (5) we have

$$\begin{aligned} \lambda(\epsilon, N) &\approx \left(\frac{\epsilon}{\pi d^2}\right) N(N\tilde{R}\log_e 2)^{-\frac{2}{\alpha}} \\ &= \left(\frac{\epsilon}{\pi d^2}\right) \tilde{R}\log_e 2^{-\frac{2}{\alpha}} N^{(1-\frac{2}{\alpha})}. \end{aligned}$$

For any path-loss exponent  $\alpha > 2$ , the maximum intensity of transmissions monotonically increases with the number of sub-bands  $N$  as  $N^{(1-\frac{2}{\alpha})}$ , i.e., *using more sub-bands with higher spectral efficiency leads to an increased transmission capacity*, as long as the linear approximation to  $\beta(N)$  remains valid. The key reason for this behavior is the fact that transmission capacity scales with the SINR threshold as  $\beta^{-\frac{2}{\alpha}}$ , which translates to  $N^{-\frac{2}{\alpha}}$  in the low spectral efficiency regime.

As  $N\tilde{R}$  increases, the linear approximation to  $\beta(N)$  becomes increasingly inaccurate because  $\beta(N)$  begins to grow *exponentially* rather than linearly with  $N$ . In this regime, the SINR cost of increasing spectral efficiency is extremely large. For example, doubling spectral efficiency requires doubling the SINR *in dB units* rather than in linear units. Clearly, the benefit of further increasing the number of sub-bands is strongly outweighed by the SINR cost.

## V. NUMERICAL RESULTS AND DISCUSSION

In Figure 2, the maximum density of transmissions is plotted as a function of  $N$  for two different spectrum utilizations  $\tilde{R}$  for a network with  $\alpha = 4$ ,  $d = 10$  m, and an outage constraint of  $\epsilon = 0.1$ . The bottom set of curves correspond to a relatively high utilization of  $\tilde{R} = 0.5$  bps/Hz, while the top set corresponds to  $\tilde{R} = 0.25$  bps/Hz. Each set of three curves correspond to the approximation from (2):  $\lambda(\epsilon, N) \approx N \left(\frac{\epsilon}{\pi d^2}\right) \beta(N)^{-\frac{2}{\alpha}}$ , numerically computed values

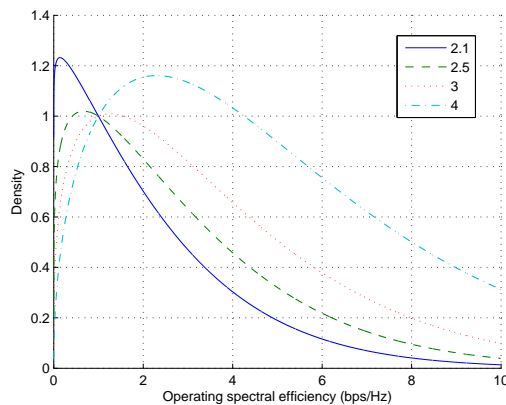


Fig. 3. Density vs. Spectral Efficiency

of  $\lambda(\epsilon, N)$  for  $SNR = \infty$ , and numerically computed values for  $SNR = 20$  dB. For both sets of curves, notice that the approximation, based on which the optimal value of  $N$  was derived, matches almost exactly with the numerically computed values. Furthermore, introducing noise into the network has a minimal effect on the density of transmissions.

For a path loss exponent of 4, evaluation of (8) yields an optimal spectral efficiency of 2.3 bps/Hz. When  $\tilde{R} = 0.25$ , this corresponds to  $N^* = \frac{2.3}{0.25} = 9.2$  and  $N = 9$  is seen to be the maximizing integer value. When  $\tilde{R} = 0.5$ , we have  $N^* = 4.6$  and  $N = 5$  is the optimal integer choice. Note that there is a significant penalty to naively choosing  $N = 1$ : for  $\tilde{R} = 0.25$  this leads to a factor of 2 decrease in density, while for  $\tilde{R} = 0.5$  this leads to loss of a factor of 1.5.

### A. Sensitivity to Spectral Efficiency

In addition to deriving the optimal spectral efficiency, it is also important to understand the sensitivity to this optimal. In Fig. 3 the quantity  $b(2^b - 1)^{-\frac{2}{\alpha}}$  (which multiplied by  $\frac{\epsilon}{\pi d^2 \tilde{R}}$  is the actual density) is plotted versus the spectral efficiency  $b$  for a few different values of  $\alpha$ . When  $\alpha$  is close to 2, a severe penalty is paid for not operating in the wideband regime ( $b \approx 0$ ). If  $\alpha$  is on the order of 3 or 4, the density  $b(2^b - 1)^{-\frac{2}{\alpha}}$  is rather peaky and a significant penalty is incurred for choosing  $b$  either too small or too large.

Perhaps the most interesting point to notice is that every curve passes through  $(1, 1)$ , because  $b(2^b - 1)^{-\frac{2}{\alpha}} = 1$  for  $b = 1$  and *any*  $\alpha$ . The choice  $b = 1$  is sub-optimal for every path loss exponent except for one particular value close to 3, but for reasonable path loss exponents (between 2 and 4) the optimal  $b^*(2^{b^*} - 1)^{-\frac{2}{\alpha}}$  is not much larger than one. Therefore, not much density is lost by choosing  $b = 1$  rather than  $b^*$ . As a result,  $b = 1$  (or  $\beta = 0$  dB) is a very useful robust operating point that can be used when the path loss exponent is not precisely known or when it varies throughout the network.

## VI. INFORMATION DENSITY

An interesting *information density* interpretation can be arrived at by plugging in the appropriate expressions for the

maximum density of transmissions when the number of sub-bands is optimized. By plugging in the optimal value of  $N$  (and ignoring the integer constraint on  $N$ , which is reasonable when  $\tilde{R}$  is considerably smaller than one) we have:

$$\lambda^*(\epsilon) \approx \left(\frac{\epsilon}{\pi d^2}\right) \frac{1}{\tilde{R}} b^*(2^{b^*} - 1)^{-\frac{2}{\alpha}} \quad (10)$$

where  $b^*$  is defined in (8) and the quantity  $b^*(2^{b^*} - 1)^{-\frac{2}{\alpha}}$  is denoted as the density constant in Fig. 1.

From this expression we can make a number of observations regarding the various parameters of interest. First note that density is directly proportional to outage  $\epsilon$  and to the inverse of the square of the range  $d^{-2}$ . Thus, doubling the outage constraint leads to a doubling of density, or inversely tightening the outage constraint by a factor of two leads to a factor of two reduction in density. The quadratic nature of the range dependence implies that doubling transmission distance leads to a factor of four reduction in density. Perhaps one of the most interesting tradeoffs is between density and rate: since the two quantities are inversely proportional, doubling the rate leads to halving the density, and vice versa.

If we consider the product of density and spectral utilization, we get a quantity that has units bps/Hz/m<sup>2</sup>:

$$\lambda^*(\epsilon)\tilde{R} \approx \left(\frac{\epsilon}{\pi d^2}\right) b^*(2^{b^*} - 1)^{-\frac{2}{\alpha}} \quad (11)$$

This quantity is very similar to the *area spectral efficiency* (ASE) defined in [8]. In our random network setting, the ASE is inversely proportional to the square of the transmission distance, which is somewhat analogous to cell radius in a cellular network, and is directly proportional to the outage constraint. Since the quantity  $b^*(2^{b^*} - 1)^{-\frac{2}{\alpha}}$  does not vary too significantly with the path-loss exponent (see Fig. 1) for  $\alpha$  between 2 and 5, we see that ASE and path-loss exponent are not very strongly dependent. Perhaps most interesting is the fact that the ASE does not depend on the desired rate: a random network can support a low density of high rate transmissions, a high density of low rate transmissions, or any intermediate point between these extremes.

## VII. GENERAL INTERPRETATION

In this section we describe the general tradeoff between bandwidth and spectral efficiency/SINR in an interference-limited network. Consider a transmitter that wishes to convey a packet consisting of  $B$  bits to a receiver located a distance  $d$  meters away. Assuming that transmission power is fixed, the transmitter has two parameters to decide upon: bandwidth  $W$  and time  $T$ . The choice of these two parameters determine the operating spectral efficiency  $b = \frac{B}{WT}$  bits/sec/Hz, as well as the operating SNR  $\beta = 2^b - 1 = 2^{\frac{B}{WT}} - 1$ . A large bandwidth-time product  $WT$  corresponds to a small spectral efficiency (i.e., wideband), and vice versa.

If interference is treated as noise, a necessary but not sufficient condition for successful transmission is that no other transmission occur on the same bandwidth-time within a distance  $d\beta^{\frac{1}{\alpha}}$  of the receiver. Therefore, the

bandwidth-time-area consumed by a transmission is:

$$WT(\pi d^2 \beta^{\frac{2}{\alpha}}) = \pi d^2 B \frac{1}{b} (2^b - 1)^{\frac{2}{\alpha}}.$$

The same metric is considered in [9] and specific coding and modulation formats are evaluated, but no general analysis is performed.

In order to maximize the density of transmissions, the bandwidth-time-area product in (12) should be minimized. In terms of  $b$ , this corresponds to:

$$\max_{b>0} b(2^b - 1)^{-\frac{2}{\alpha}}.$$

This maximization is clearly identical to the optimization in Section IV-B, and thus the optimal spectral efficiency is also given by (8). Thus, the optimal spectral efficiency derived earlier has a rather general interpretation in the context of interference-limited networks.

## VIII. CONCLUSION

In this work we studied bandwidth-SINR tradeoffs in ad-hoc networks and derived the optimal operating spectral efficiency, which was shown to be a function only of the path loss exponent. A network can operate at this optimal point by dividing the total bandwidth into sub-bands sized such that each transmission occurs on one of the sub-bands at precisely the optimal spectral efficiency.

The key takeaway of this work is that an interference-limited ad-hoc network should operate in neither the wideband (power-limited) nor high-SNR (bandwidth-limited) regimes, but rather at a point between the two extremes because this is where the optimal balance between multi-user interference and bandwidth is achieved. Although we considered a rather simple network model, we believe that many of the insights developed here will also apply to more complicated scenarios, e.g., wideband fading channels and networks in which some degree of local transmission scheduling is performed.

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