Comparative Performance Evaluation of MAC Protocols in Ad Hoc Networks with Bandwidth Partitioning

(Invited Paper)

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***Abstract*—This paper considers the performance of the MAC protocols ALOHA and CSMA in wireless ad hoc networks, where the total system bandwidth may be divided into smaller subbands. In the network model used, the arrival of users/packets follows a Poisson point process, communication between nodes is continuous in time, selection of a subband to transmit across is made randomly at each transmitter, and the outage assessments made in the network are based on SINR measurements. Accurate bounds on the probability of outage for the MAC protocols are derived, and evaluated with respect to the number of subbands. It is observed that there exists an optimal number of subbands for each protocol, for which the probability of outage is minimized. For ALOHA, we obtain an analytical expression for this optimal value, while in CSMA, the optimal value is observed through simulations. Furthermore, we improve the performance of CSMA by introducing channel sensing across all subbands, in order to decrease the probability that a packet is in outage upon arrival. The obtained results are used to compare the performance of the two MAC protocols. Finally, we also evaluate the performance of our network in terms of sum capacity.**

I. Introduction

In many ad hoc wireless systems, such as the IEEE 802.11 and 802.16 standards family, there is great need for efficient allocation of resources. As interference is typically the major issue limiting the performance in such systems, multiple access control (MAC) protocols are often applied to improve the communication in the network. Two of the most popular MAC protocols are ALOHA and Carrier-Sensing Multiple Access (CSMA), on which we focus in this paper.

We consider a network model in which transmitter nodes are randomly located according to a 2-dimensional Poisson point process (PPP) with a specified spatial density, and packets arrive randomly in time according to a 1-dimensional PPP with a certain density. In order to derive precise results, we focus exclusively on single-hop communication, and assume that each transmitter (TX) wishes to communicate with a receiver (RX) a certain distance away from it. All multiuser interference is treated as noise, and our model uses the SINR to evaluate the performance (as measured in terms of outage probability) of the communication system. The only source of randomness in the model is thus in the location of nodes and concurrent transmissions, which allows us to focus on the relationships between transmission density, SINR threshold, outage probability, and the choice of MAC protocol.

Based on this model, we ask the following questions: Given a fixed system bandwidth and requested transmission rate in a wireless ad hoc network, (a) how many subbands should the bandwidth be divided into, in order to minimize the probability of outage for a given requested rate per link, and (b) what is the benefit of introducing sensing across subbands compared to having a random selection of a subband?

A. Related Work

A number of different works have analyzed ALOHA using a Poisson model for TX locations [6] [7]. Perhaps the closest work is that of Hasan and Andrews [5], where success probability of ALOHA and CSMA was analyzed for a similar spatial model, assuming that a scheduling mechanism creates an interference-free *guard zone* (i.e., circle) around the RX. Moreover, some works have compared the ALOHA and CSMA protocols in terms of transport capacity [6] [11], throughput [11] [12], spatial reuse [7], and transmission delay [12]. All the above-mentioned performance metrics can be seen to be in some way related to SINR measurements.

Recently obtained results give us mathematical expressions for lower bounds on the outage probability of ALOHA and CSMA [1]. However, beyond this, under the assumption of our Poisson based model with random location of nodes, neither unslotted ALOHA nor CSMA appear to have been analyzed in detail. The IEEE standards 802.3 and 802.11, as well as many sensor networks, apply various versions of CSMA to efficiently share the medium. Furthermore, some of the works done on the performance of MAC protocols, such as [11], consider deterministic channel access schemes, which preclude the occurrences of outages. In order to best model the behavior of a distributed ad hoc network at the MAC layer, a stochastic SINR requirement must be used, as is the case in our model.

The concept of bandwidth partitioning is a well-studied topic. In the context of decentralized networks [9], the transport capacity of a random access network is maximized by jointly optimizing rate, TX-RX distance, and density. In [10], an ad hoc network with a high density of interfering TX-RX

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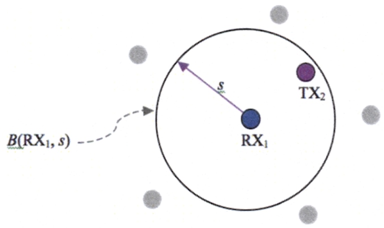

Fig. 1. When at least one interferer TX_2 falls within a distance s away from RX_1, i.e., within $B(\mathrm{RX}_1, s)$, it causes outage for RX_1.

pairs is analyzed, but no spatial model is used and only fading is considered. In [2], the number of simultaneous transmissions in a multiuser decentralized network is maximized by optimizing the number of subbands the system bandwidth is divided into. In these works, however, only ALOHA-like protocols are considered. In this paper, we take a step further to also include CSMA-like protocols when optimizing the performance of a network by means of bandwidth partitioning. Also, we are concerned with our packets being received correctly, rather than increasing the information flow in the system. Outage probability is thus the most appropriate performance metric.

II. System Model

Our system model considers an ad hoc network, where nodes are distributed randomly in space. Specifically, TXs are located on an infinite 2-D plane according to a homogeneous PPP with spatial density λ^s [nodes/m^2], an assumption that is reasonable particularly in a network with substantial mobility or indiscriminate node placement, such as a very dense sensor network. Packets arrive at each TX according to an independent 1-D PPP with density λ^t [packets/sec], and are sent with a constant power ρ to the intended RX, which is assumed to lie a *fixed* distance R away. The fixed distance between TX and RX is clearly not a natural assumption, but it has been rigorously shown in [3] that variable transmit distances do not result in fundamentally different performances. Moreover, the whole network with a fixed R could be viewed as a snapshot of a multi-hop wireless network, and R can then be viewed as the bounded average inter-relay distance designed by the routing protocol. Furthermore, each packet has a fixed duration T, meaning that at each time instant, the density of TXs that have received packets during the last T seconds is equal to $\lambda = \lambda^s \lambda^t T$.

We may also consider this ad hoc network from another point of view. Instead of fixing the locations of the TXs prior to the packet arrivals, we let packet arrivals be the random variable that we consider, following a PPP in time. Upon the arrival of each packet, we assign it to a TX-RX pair, which is then placed randomly on the plane. This alternative network model, which was introduced in [1], and which still entails a Poisson distribution of nodes in both space and packets in time, greatly simplifies the analytical work, as it allows us to consider a single random process describing both the temporal and spatial variation of the system.

For the channel model, only path loss attenuation effects (with exponent $\alpha > 2$) are considered, i.e. additional channel effects such as shadowing and fast fading are ignored, and the channel is considered to be constant for the duration of a transmission. Each RX potentially sees interference from all TXs, and these independent interference powers are added to the channel noise η, to result in a certain SINR at each RX. If this SINR falls below the required SINR threshold β at any time during the packet transmission, the packet is received in *outage*. With an outage probability constraint ϵ, we demand that:

$$\Pr\left(\frac{\rho R^{-\alpha}}{\eta + \sum_i \rho |r_i|^{-\alpha}} \leq \beta\right) \leq \epsilon, \tag{1}$$

where $\Pr$ denotes probability and r_i is the distance between the node under observation and the i-th interfering TX.

Moreover, the system has a fixed total bandwidth W, and each TX wishes to send information at a *fixed* requested rate R_{req}, which is assumed to be the same for all users. The bandwidth is to be divided into N subbands, where $N = 1$ indicates that there is no bandwidth partitioning, meaning that all users apply the whole bandwidth W, and thus producing the same results obtained in [1]. For $N > 1$, we assume that each user can communicate on only one out of the N subbands at a time. This then results in changes in the effective density of interferers, the noise in the system, the area of the guard zone around each node, and thereby the outage probability for both ALOHA and CSMA. Whereas the requested rate remains constant regardless of the value of N, the *required spectral efficiency* of the system, which we define as NR_{req}/W, will change with N, resulting in dependence of β on N.

In the case of unslotted ALOHA, each transmission starts as soon as the nodes are placed on the plane, regardless of the channel condition. *Slotted ALOHA* improves performance by removing partial outages, but such a system would require synchronization. In the CSMA protocol the incoming TX listens to the channel at the beginning of the packet, and if the measured SINR is below β, it drops its packet. No retransmissions are applied in our model.

III. Outage Probability as Performance Metric

As has been shown by previous works, which do not allow for bandwidth paritioning [1] [6], the outage probabilities of both ALOHA and CSMA are dependent on the value of the SINR threshold β. This dependence is explicitly existent through the radius, s, of the guard zone, $B(\mathrm{RX}_i, s)$, defined to be the distance between the RX under observation and one single closest interferer that would cause the SINR to fall just below the threshold β. This is given by:

$$s = \left(\frac{R^{-\alpha}}{\beta} - \frac{\eta}{\rho}\right)^{-\frac{1}{\alpha}} \tag{2}$$

Consider the area of the guard zone $B(\mathrm{RX}_1, s)$, which is a circle of radius s around the RX under observation, RX_1, as illustrated in Fig. 1. One situation that would cause RX_1 go in outage, even if there are no interferers inside the guard zone, is if the accumulation of powers from all the interfering nodes

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outside $B(\mathrm{RX}_1, s)$ results in the received SINR at RX_1 to be below the threshold β. Another situation that might also lead to the reception of a packet in outage is if at least one active TX, other than RX_1's own transmitter, TX_1, falls within $B(\mathrm{RX}_1, s)$ any time during the packet transmission. This latter case gives the *lower bound* to the probability of outage [6], because as the number of interferers exceeds 1, the probability of outage also increases. Based on previous works [6], which have shown that the upper and lower bounds on the outage probability can be fairly tight around the actual outage probability, in this paper we only consider the the lower bound as proposed in [6].

Def. 1: A packet transmission is considered to be in outage if the achievable rate of transmission R_i for link i is less than the requested rate of transmission R_{req}.

Based on the Shannon capacity formula for AWGN channel, $R_i = W \log(1 + \mathrm{SINR}_i)$, we thus have that:

$$\begin{aligned} P_{out} &= \Pr\left[R_i \leq R_{req}\right] \\ &= \Pr\left[W \log(1 + \mathrm{SINR}_i) \leq R_{req}\right] \qquad (3) \\ &= \Pr\left[\mathrm{SINR}_i \leq 2^{R_{req}/W} - 1\right]. \end{aligned}$$

Hence, Def. 1 is equivalent to the SINR falling below the threshold $\beta = 2^{R_{req}/W} - 1$.

A. Effect of Bandwidth Partitioning on SINR Threshold

Dividing the system bandwidth into N subbands greatly affects the achievable transmission rate per link for a given SINR threshold. Having N subbands, the Shannon capacity formula gives us: $R_i = \frac{W}{N}\log(1+\mathrm{SINR})$. Using the derivations of Equation (3), and rearranging, we obtain:

$$\beta(N) = 2^{NR_{req}/W} - 1, \qquad (4)$$

where NR_{req}/W is the required spectral efficiency of the system. Moreover, due to the dependence of s on β in Equation (2), we now have that s is also a function of N. In a strictly interference-limited system, we may set the noise power η in (2) to zero, resulting in $s(N) = R\beta(N)^{1/\alpha}$. This simplification is used for analytical tractability in later calculations.

In the following two subsections, we first assume that each TX makes a *random* selection of one subband to transmit its packet over. In subsection III-D, we improve the outage probability by introducing sensing across subbands before a random selection of one subband is performed over all the subbands that entail a measured SINR above the threshold $\beta(N)$. Due to the random selection of subbands, we may still assume that TXs are Poisson distributed on the plane, an assumption that the simulation results confirm to be reasonable.

B. Bandwidth Partitioning for ALOHA

In the ALOHA protocol, packets are transmitted at fixed R_{req} immediately upon arrival, regardless of whether or not $\mathrm{SINR}_i > \beta(N)$. In *slotted ALOHA*, which has been analyzed in numerous works, e.g. [5] and [8], TXs can only start their packet transmissions at the beginning of the next time slot after the packets have been formed. Thus there is no partial overlap of transmitted packets, something that is intuitively expected to decrease the outage probability, compared to the continuous-time scenario. However, as mentioned before, this is at the expense of a need for synchronization.

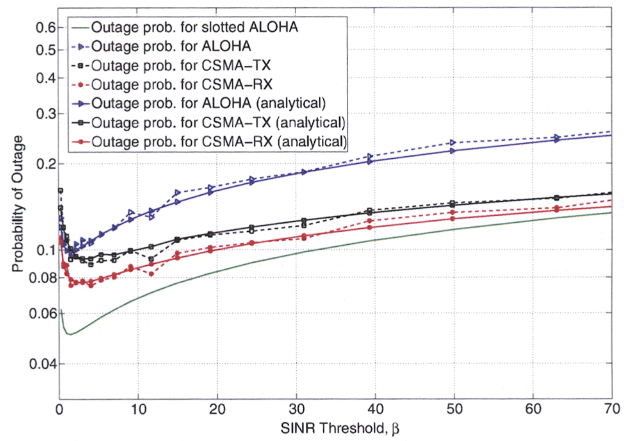

Fig. 2. Sensitivity of the outage probability with respect to the SINR threshold $\beta(N)$ for fixed $\lambda = 0.005$.

Theorem 1: The lower bounds on the probability of outage for slotted and unslotted ALOHA with the system bandwidth divided into N subbands are, respectively:

$$P_{out}^{LB}(\text{Slotted ALOHA}) = 1 - e^{-\lambda\pi s^2(N)/N} \qquad (5)$$

$$P_{out}^{LB}(\text{Unslotted ALOHA}) = 1 - e^{-2\lambda\pi s^2(N)/N} \qquad (6)$$

where $s(N)$ is given by (2) with $\beta = \beta(N)$ given by (4).

Outline of proof for Theorem 1: The formulas for the bounds on the probability of outage for slotted and unslotted ALOHA without bandwidth partitioning were obtained in [1]. Equations (5) and (6) are derived based on the probability that an active TX is placed inside the guard zone of the RX under consideration, as illustrated in Fig. 1. For the unslotted system, due to the existence of partial overlap of packets, we require that there are no interfering TXs inside $B(\mathrm{RX}_1, s)$ during the period $[-T, T]$, which corresponds to two packet durations, hence the 2 in the exponent of Equation (6). Moreover, the bandwidth partitioning in N subbands results in the density of nodes in each subband being reduced by a factor N, while the SINR threshold is increased based on Equation (4).

The increase in $\beta(N)$ with N, which is necessary to uphold a constant R_{req} when the available bandwidth is decreased, also results in an increase in $s(N)$. This means that a greater area around the RX must be free of interferers in order to avoid receiving a packet in outage. From Equation (4), we have that for low values of the spectral efficiency, $\beta(N)$ is approximately a linear function of N for both slotted and unslotted ALOHA.

Writing out equations (5) and (6) with Taylor expansions, and using approximations based on small values of the exponent of e, we observe that slotted ALOHA outperforms unslotted ALOHA in terms of outage probability by approximately a factor of 2. This is also evident from Fig. 2, and is indeed consistent with the results obtained from the conventional

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model for the slotted and unslotted ALOHA protocols [11]. The analytical expressions for the outage probabilities are also plotted in Fig. 2, for a fixed transmission density, and are shown to follow the simulation results tightly. Note that for very low values of $\beta(N)$, the outage probability increases as $\beta(N)$ decreases. This may understood by observing that as $\beta(N) \to 0$, so does the required spectral efficiency through Equation (4). This would be equivalent to saying that R_{req} goes to 0, which would by definition (Def. 1) result in the outage probability going to 1.

Moreover, from Fig. 2, we also observe that there exists an optimal value of N for which the probability of outage is minimized. To find this, we assume that N can take any value (i.e., not only discrete integer values), and then differentiate P_{out} with respect to N. Setting the derivative equal to 0 and solving for N_{opt}, results, after some manipulations, in:

$$\begin{aligned} N_{opt} &= \frac{W}{2R_{req}\ln(2)}\left[\alpha + 2W_0\left(-\frac{1}{2}\alpha e^{-\alpha/2}\right)\right] \quad (7) \\ &= \frac{W}{2R_{req}\ln(2)}\left[\alpha + 2\sum_{n=1}^{\infty}\frac{(-n)^{n-1}}{n!}\left(-\frac{1}{2}\alpha e^{-\alpha/2}\right)^n\right], \end{aligned}$$

where $W_0(\cdot)$ is the Lambert function. Note that N_{opt} is the same for both slotted and unslotted ALOHA. This is due to the fact that the number of active nodes, and thus interferers, in both systems is the same. Note also that the optimal number of subbands is dependent only on α and R_{req}/W. For example, if $\alpha = 3$ and $R_{req}/W = 1/3$, which are the parameters used in our simulations, we obtain both from the graphs and from Equation (7) that the closest integer value of N_{opt} is 4.

C. Bandwidth Partitioning for CSMA

In the CSMA protocol, a TX backs off if the accumulated interference from all other active TXs results in a measured SINR that is below β at the *beginning* of the packet. The probability of this is called *backoff probability*, P_b, and it is equivalent to the probability that a TX drops its packet, since our system model entails no retransmissions. If the TX decides to transmit, but the SINR at the RX is below $\beta(N)$ any time along the packet duration, the packet is received in outage.

Based on prior work [1], we apply the analytical expressions for the total outage probability for CSMA, and incorporate bandwidth partitioning into the equations through $\lambda(N)$, $\beta(N)$, and $s(N)$. Considering the conventional CSMA protocol where the *transmitter* senses the channel and decides whether or not to transmit, this means that if the incoming TX falls within a distance $s(N)$ away from an already active TX on the plane, this new TX backs off. Note that because of the backoff property of CSMA, the number of TXs on the plane no longer follows an exact PPP. Nevertheless, as an approximation, we assume that nodes are still Poisson distributed, and our simulation results show that this assumption is in fact reasonable.

Theorem 2: The probability of outage for CSMA with *transmitter sensing*, CSMA-TX, is given by:

$$\begin{aligned} &P_{out}^{LB}(\text{CSMA}) = P_b + (1-P_b)P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \quad (8) \\ &+P_b\left[1-P_{out}^{LB}(\text{CSMA}|\text{no backoff})\right]\left[1-P_{out}^{LB}(\text{RX beg.}|\text{backoff})\right], \end{aligned}$$

where P_b is the probability of backoff and is given in terms of the Lambert function as:

$$P_b = 1 - \frac{W_o\left(\lambda\pi s(N)^2/N\right)}{\lambda\pi s(N)^2/N}. \quad (9)$$

Furthermore, $P_{out}^{LB}(\text{CSMA}|\text{no backoff})$ is the probability that a packet is received in outage, given an active TX-RX pair:

$$\begin{aligned} &P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \quad (10) \\ &= \int_{(s-R)^2}^{s^2}\left[1 - \frac{1}{\pi}cos^{-1}\left(\frac{d^2+R^2-s(N)^2}{2Rd}\right)\right]\pi\frac{\lambda}{N}e^{-\pi\frac{\lambda}{N}d^2}d(d^2). \end{aligned}$$

Finally, $P_{out}^{LB}(\text{RX beg.}|\text{backoff})$ is the probability that the closest interfering TX, which is inside $B(\text{TX}_1, s(N))$, is also inside $B(\text{RX}_1, s(N))$. That is:

$$\begin{aligned} &P_{out}^{LB}(\text{RX beg.}|\text{backoff}) \\ &= \frac{2}{\pi}cos^{-1}\left(\frac{R}{2s(N)}\right) - \frac{R}{\pi s(N)}\sqrt{1-\left(\frac{R}{2s(N)}\right)^2}. \quad (11) \end{aligned}$$

Outline of proof for Theorem 2: The derivations of the above formulas are based on the requirement that the guard zone around the active TX under consideration is free of interferers both upon the arrival of the packet, as well as during its transmission. We do not go into the details of how these formulas are derived; the interested reader may refer to [1]. However, what is different in these equations as compared to those in [1], is that the density λ is now replaced by λ/N, and that the guard zone radius s is a function of N.

It was also shown in [1] that by letting the *receiver* sense the channel and inform its TX over a control channel of whether or not to initiate its transmission, the outage probability of CSMA may be reduced considerably. Such sensing at the RX adds an extra consideration to the outage probability, namely the relative position of RX_2 with respect to TX_1 and TX_2.

Theorem 3: The probability of outage for CSMA with *receiver sensing*, CSMA-RX, is given by:

$$P_{out}^{LB}(\text{CSMA}) = P_b + (1-P_b)P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \quad (12)$$

where the probability of backoff, P_b, is the same as in the case of TX sensing, given in Equation (9). $P_{out}^{LB}(\text{CSMA}|\text{no backoff})$ is the probability that an ongoing packet transmission is received in outage, and is given by:

$$\begin{aligned} &P_{out}^{LB}(\text{CSMA}|\text{no backoff}) \quad (13) \\ &= \int_0^{s^2}\int_{\alpha(d)}^{\gamma(d)}\frac{1}{2\pi}P(\text{active}|d,\phi)\ \pi\frac{\lambda}{N}e^{-\pi\lambda d^2/N}d\phi\ d(d^2) \end{aligned}$$

where $P(\text{active}|d,\phi)$, $\alpha(d)$, and $\gamma(d)$ are given as:

$$P(\text{active}|d,\phi) = 1 - \frac{1}{\pi}cos\left(\frac{d^2+2R^2-s(N)^2-2Rdcos\phi}{2R\sqrt{d^2+R^2-2Rdcos\phi}}\right)$$

$$\alpha(d) = cos^{-1}\left(\frac{d^2+2Rs(N)-s(N)^2}{2Rd}\right) \quad ; \quad \gamma(d) = 2\pi - \alpha(d).$$

As mentioned earlier, the detailed derivations of these expressions may be found in [1], with the difference that the

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density is now reduced by a factor N and the guard zone is increased with N, according to equations (2) and (4).

Fig. 2 shows the simulated and analytical results for the outage probability of CSMA with respect to the number of subbands N. Firstly, the figure confirms that the obtained analytical results follow the simulation results tightly. Secondly, we note that the outage probability of CSMA is less than that of ALOHA for almost all values of N. This is primarily due to the fact that CSMA reduces the overall interference level by backing off packets that are certain of being unsuccessful. Finally, we see that the optimum value of N, for which the probability of outage is minimized, is larger than that of ALOHA; for CSMA-TX, $N_{opt} = 7$, and for CSMA-RX, $N_{opt} = 6$. These findings indicate among others that a higher rate of transmission may be obtained with CSMA than with ALOHA for a constant value of outage probability. Note, however, that for very low values of $\beta(N)$, CSMA-TX does in fact perform worse than ALOHA, as was also concluded in [1]. This is because CSMA-TX backs off in cases when its RX would have received the packet without errors. It should also be noted that for CSMA-RX, we have not considered the "stealing" of resources that the feedback channel requires.

D. Sensing Across Subbands

In the previous sections, we assumed that each TX randomly selects one subband to transmit over for each new packet. In this subsection, we improve the performance of the multi-band system by allowing the nodes to sense the channel conditions across all subbands first, to locate those (if any) where the SINR is above the required threshold $\beta(N)$. Then the sensing node makes a random selection of one subband from these chosen subbands, over which it initiates its packet transmission. Due to this random selection of a subband among those that have a measured SINR above β, we may still assume that the number active of nodes within each subband follows a PPP. The additional channel information across all subbands thus acquired at the sensing node, results in a reduction in the outage probability.

Once a packet transmission has been initiated, the probability that the packet will be received in outage any time during its transmission is the same as before (equations (10) and (13)). However, the backoff probability is changed. In order for a packet to back off, it has to be in outage simultaneously in all the N subbands of the system upon arrival. That is:

$$P_{bN} = \left[1 - \frac{W_o\left(\lambda\pi s(N)^2/N\right)}{\lambda\pi s(N)^2/N}\right]^N, \qquad (14)$$

where $W_0(\cdot)$ is the Lambert function as expressed in (7).

The total outage probability for CSMA-RX is found based on Equation (12) with P_b replaced by P_{bN}. The total outage probability for CSMA-TX, however, is given by:

$$\begin{aligned} &P_{out}^{LB}(\text{CSMA}) \qquad (15)\\ &= P_{bN} + (1 - P_{bN})P_{out}^{LB}(\text{CSMA}|\text{no backoff}) + \sqrt[N]{P_{bN}}\\ &\cdot\left[1 - P_{out}^{LB}(\text{CSMA}|\text{no backoff})\right]\left[1 - P_{out}^{LB}(\text{RX beg.}|\text{backoff})\right], \end{aligned}$$

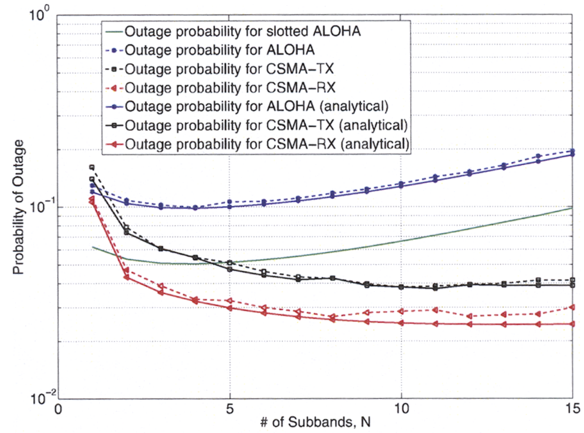

Fig. 3. Outage probability of ALOHA and CSMA with respect to the number of subbands N for $\lambda = 0.005$, when we have sensing across subbands.

where P_{bN} is given in (14), and $P_{out}^{LB}(\text{CSMA}|\text{no backoff})$ and $P_{out}^{LB}(\text{RX beg.}|\text{backoff})$ are, as before, given by (10) and (11). Note that the only difference between equations (8) and (15) is that P_b is replaced by P_{bN}, and in the last term, P_b has been replaced by $\sqrt[N]{P_{bN}}$, which are essentially equal. The latter replacement is because the last term of Equation (15) expresses the probability that the RX is in outage upon arrival, once the TX has chosen a subband to transmit over. Hence, this is the same as the probability that the RX is in outage in *one* subband, hence $\sqrt[N]{P_{bN}}$.

In Fig. 3, the outage probability of CSMA-TX and CSMA-RX with capability to sense across all subbands, is plotted as a function of N. As expected, we witness a significant improvement compared to CSMA without sensing across subbands. As N increases, this advantage becomes more evident. E.g., for $N = 13$ (for which the outage probability of CSMA-RX in Fig. 3 is minimized), the introduction of sensing across subbands, reduces the outage probability of CSMA-TX by approximately 65%, and that of CSMA-RX by 75%. Measuring around $N = 6$, for which the outage probability of CSMA-RX is minimized in the case of no sensing, the introduction of sensing across subbands reduces outage probability by about 34% for CSMA-TX, and about 53% for CSMA-RX. Moreover, the benefit of using CSMA-RX over CSMA-TX also increases when we introduce sensing across subbands. E.g., at $N = 13$, this advantage is approximately tripled when having sensing, compared to when subbands are selected on a random basis.

IV. Sum Capacity as Performance Metric

In this section we consider the performances of ALOHA and CSMA in terms of the *sum capacity* of the network. With bandwidth partitioning, the capacity of the system is:

$$C = \sum_{j\in F} \frac{W}{N} \log\left(1 + \frac{\rho R^{-\alpha}}{\eta + \sum_{i\in I_j} \rho |r_{ij}|^{-\alpha}}\right), \qquad (16)$$

where F is the set of all transmitted packets over the duration of all transmissions (which is equivalent to the number of active transmissions at a snapshot of the network), and I_j is the

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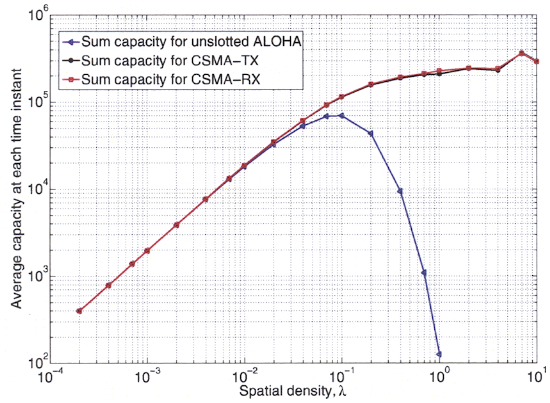

Fig. 4. Sum capacity per time instant with respect to density for $N = 20$.

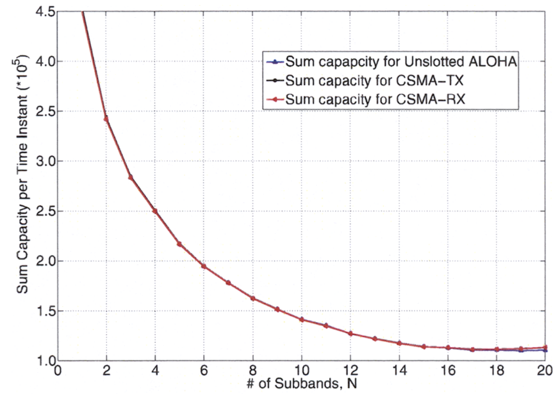

Fig. 5. Sum capacity per time instant with respect to N for $\lambda = 0.005$.

set of all interferers for RX_j over one subband. The difference between the MAC protocols becomes evident through I_j. Fig. 4 shows the outage probability of ALOHA and CSMA with respect to the density λ for $N = 20$. In the case of ALOHA, the sum capacity increases until the density of interferers exceeds a critical value, above which P_{out} decreases rapidly towards 0. For CSMA, however, the sum capacity continues increasing past the critical density, although at a lower rate. This is because the capacity metric does not care whether or not a packet is received in outage for a given transmission scheme; it is only concerned with the maximum amount of information that could be transmitted if ideal coding were available.

Fig. 5 shows the effect of bandwidth partitioning on the sum capacity. As the number of subbands N increases, the sum capacity of the system decreases. This means that if increasing the total sum capacity of the system in an information-theoretic sense is the objective, and not the successful reception of packets, then $N = 1$ is the optimal number of subbands. Note, however, that achieving this capacity would require rate-adaptive transmission, whereas for the schemes we have considered, all transmitting nodes would use rate R_{req}.

V. Conclusion and Future Work

In this paper we have considered the performance of slotted and unslotted ALOHA and CSMA with TX and RX sensing, in terms of probability of outage and sum capacity. Our ad hoc network model entails a continuous-time communication system where TX-RX pairs are randomly placed on a 2-D plane, and packets arrive based on a 1-D Poisson point process. Dividing the system bandwidth into N subbands, we derived analytical expressions for the outage probability both for the case when a subband is selected randomly, and the case when a subband is selected among those having a measured SINR above $\beta(N)$. Moreover, we analytically found the optimal number of subbands that minimizes the outage probability of ALOHA. When we allow for sensing across bands, we see that the outage probability is reduced and the optimal value of N is increased, meaning that a higher rate of transmission per link may be obtained for a given outage probability. However, the sum rate of the network is reduced as there will be fewer concurrent transmissions.

For future work, we will extend the results obtained in this paper to include packet transmission over multiple subbands. Also, we wish to compare the obtained results to the case where the subband chosen for transmission is no longer based on a random selection (which was also the case when we had sensing across subbands), but rather based on selecting the subband that entails the best channel condition. Moreover, we wish to find analytical expressions for the optimal number of subbands in the case of CSMA. It is also of interest to add retransmissions to the model, and analyze the transmission rate and delay of packets.

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