COMPARING RLS AND LMS ADAPTIVE EQUALIZERS FOR NONSTATIONARY WIRELESS CHANNELS IN MOBILE AD HOC NETWORKS

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Abstract - This paper compares performance of finite impulse response (FIR) adaptive linear equalizers based on the recursive least-squares (RLS) and least mean square (LMS) algorithms in nonstationary uncorrelated scattering wireless channels. Simulation results, in terms of steady-state mean-square estimation error (MSE) and average bit-error rate (BER) metrics, are found for the frequency-selective Rayleigh fading wireless channel experienced in a mobile ad hoc network where nodes are lognormally shadowed from each other. For the nonstationary channel models considered, RLS is always found to outperform LMS.

Keywords - filtering, adaptive equalizers, least mean square methods, least squares methods, Rayleigh channels, simulation

I. INTRODUCTION

Nonstationary environments arise when the random process supplying the tap inputs of an adaptive filter (see Fig. 1) is nonstationary itself [1]. One instance in which this condition occurs is when the filter is employed to equalize time-varying channels, a unique phenomenon found in wireless communication environments [2]. A nonstationary process is a stochastic process whose statistics are time-varying [3]. In this paper, we consider the *channel* itself to not just be time-varying but nonstationary too. Nonstationary implies time-varying, and equalizing a nonstationary channel also involves a nonstationary environment. We consider for simplicity *first-order nonstationarity*, the mean of the random process being time-varying. This is a good model for the wireless propagation environment where a received signal experiences large fluctuations (multipath or small-scale fading) and is nonstationary for distances much larger than a wavelength since the local mean of the fading signal changes significantly as different objects become reflectors [4].

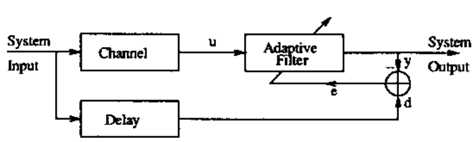

Fig. 1. Adaptive Equalizer. u is the input taps applied to the equalizer, y is the output of the equalizer, d is the desired response, and $e = d - y$ is the estimation error.

Two of the most prominent families of adaptive filtering algorithms are the LMS and RLS families. Not much literature exists on comparing LMS and RLS in realistic environments. Herein we compare the performance of linear transversal, or FIR, adaptive digital filters based on LMS and RLS, when the optimal (Wiener) filtering vector is randomly time-varying due to the nonstationary channel, in which case the algorithms must track the minimum point of the error-performance surface, as with the time-varying channel case. In this paper, the focus is on adaptive linear equalization of nonstationary uncorrelated scattering wireless channels. The speed and stability with which adaptation takes place is regulated by the LMS step-size parameter μ and the RLS forgetting factor β, where faster adaptation corresponds to larger μ and smaller β, respectively [1]. Ability to adapt is affected by the ability to track the signal as well as the inherent noisiness of the environment and of the algorithm itself. Simulation results are based on the performance metrics of steady-state MSE and average BER after adaptive equalization in the practical nonstationary wireless scenario of frequency-selective Rayleigh fading and lognormal shadowing (large-scale fading) in the context of a mobile ad hoc network.

Without requiring pre-existing infrastructure or centralized administration, a mobile wireless ad hoc network often consists of transceiver nodes communicating over multiple hops in an arbitrary topology [5], [6]. Focusing on physical layer issues, this study explores a fundamental building block of an ad hoc network, the point-to-point radio link, and extends [7] and [8] in its investigation of link performance in channels, now with delay spread and intersymbol interference (ISI). Since all nodes can move in ad hoc networks, a link can suffer from *double mobility* (both sender and receiver mobile with speeds v_1 and v_2, respectively), leading to Doppler effects that depend on *degree of double mobility* $\alpha = \frac{min(|v_1|,|v_2|)}{max(|v_1|,|v_2|)}$, where $\alpha = 1$ for *full double mobility* and $\alpha = 0$ for *single mobility* (like a cellular link) [9]. Fig. 2 shows how the Doppler power spectrum changes with α, and [9] contains analytical expressions showing that the rms Doppler spread changes by as much as 1.5 dB with α. [10] and [11] contain actual Doppler spectrum measurements for setups (fixed nodes with moving scatterers in a partially shadowed environment and inter-vehicle mobile communication in highway and other environments, respectively) that, since only somewhat related, at best suggest a heuristic experimental verification of these “ad hoc” spectra.

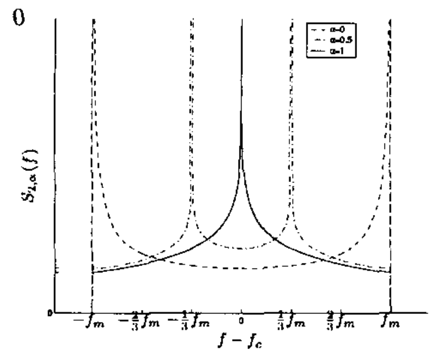

Fig. 2. Rayleigh Doppler Spectrum: Ad hoc Case, Fixed f_m

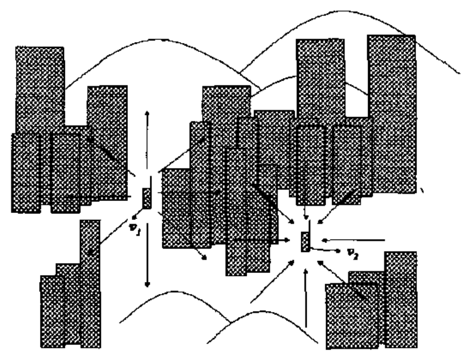

Fig. 3. A Rayleigh fading and lognormally shadowed link within a mobile ad hoc network.

This paper is organized as follows. Section II summarizes the system and channel models, and section III discusses results and concludes the paper.

II. Models

A. System Model

The RLS and LMS filter tap update algorithms are implemented as in [1] and [12], with the replica of the desired response generated locally in the receiver using training (as opposed to the decision-directed method). For convenience, we use “LMS” to refer to the slightly modified *normalized LMS* algorithm [1]. Training is used on a continuous basis to minimize error propagation and allow us to study the inherent nature of the update algorithms more closely. The input signal is delayed so the filter is centered on the finite-duration channel response.

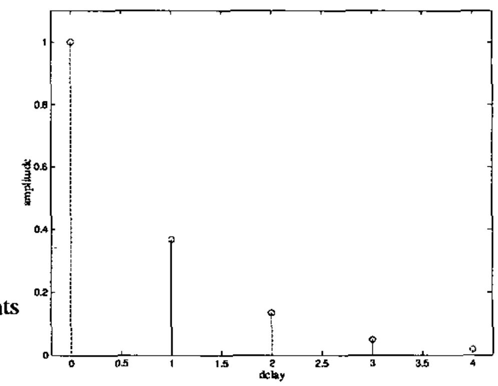

Fig. 4. Exponential power-delay profile (5 taps).

B. Channel Model

We consider the frequency-selective Rayleigh (worst-case) fading channel in an ad hoc mobile wireless network, encountered when two communicating terminals are heavily shadowed from each other (no dominant path, see Fig. 3) and signal energy arrives equally divided in angles uniformly distributed in the horizontal plane. Rayleigh fading can be accurately simulated using the Line Spectrum (LS) simulation model, defined in [7]. In order to simulate frequency-selective fading for a specific α (see Fig. 2), we generate via LS an independent Rayleigh fading gain for each channel tap. We consider two extremes, the fastest fading environment ($\alpha = 0$, hardest to track) and the slowest fading ($\alpha = 1$, easiest to track). We use a channel with 5 taps, each set initially to values given by the normalized exponential power-delay profile (see Fig. 4):

$$P(\tau) = \begin{cases} e^{-\frac{\tau}{S}}, & \tau \geq 0, \\ 0 & \text{otherwise} \end{cases}$$

where the rms delay spread $S = 5$ μs, typical of urban outdoor environments, and close to the symbol rate, $T_s = \frac{1}{R_b} = 10$ μs, necessitating the use of equalization to mitigate the ISI.

Our Rayleigh fading process is first-order nonstationary by nature since its mean is time-varying and follows a lognormal shadowing process with correlation distance of $x_c = 10$ m, assuming the “ad hoc” long-range model described in [7] and [8]. User mobility maps the spatial signal variation into a temporal variation. Thus, the extent of nonstationarity of the channel can be captured by the shadowing correlation/coherence time T_c, the interval over which statistics are roughly invariant. For the remainder of the paper, we refer to T_c as simply the “coherence time,” not to be confused with the coherence time associated with small-scale fading. When converted to seconds, $T_c = \frac{x_c}{v}$, where the effective speed $v = v_1 + v_2$ and is linearly proportional to maximum Doppler shift f_m. We simulate four different outdoor scenarios as shown in Table 1. For fixed f_m, the nonstationarity of the environment decreases as α increases, since the “small-scale fading coherence time” (inversely proportional to rms Doppler spread) increases [9]. Varying α does not change the nonstationarity of the channel but rather that of the environment, and, in the end, the statistics of the input and desired response for the adaptive filter is what matters.

We assume the distance between users is always $d = 1$ km in our shadowed urban scenario with path loss exponent

Table 1
Outdoor modes of transportation

Mode	v (mph)	T_c (symbols)
Pedestrian	4	560000
Bicycle	15	149333
Car (slow)	40	56000
Car (fast)	90	24889

of $\gamma = 4$. BPSK is the assumed modulation, and, assuming proper pulse shaping for the binary signaling, bandwidth W (kHz) equals bit rate $R_b = 100$ kbps. Transmit power $P_t = 100$ mW, and the noise samples are uncorrelated Gaussian with power $N = N_0 W = 10^{-15}$ W taking into account a typical 4 dB loss due to receiver noise figure and non-ideal carrier recovery phase noise.

III. Discussion and Conclusion

Figs. 5-6 show sample realizations for the nonstationary channel models. The channel taps vary more rapidly for smaller α, or higher Doppler spread, as expected [9]. The shape of the initial channel tap profile did not have much of an effect on results, however. BER and MSE performance of LMS and RLS over the four different coherence times (see Table 1) for different adaptive parameters (μ and β, respectively) were evaluated, and the minimum possible BER or MSE for a given value of T_c was extracted from the bowl-shaped curves of Figs. 7-10 to end up with Figs. 11-12.

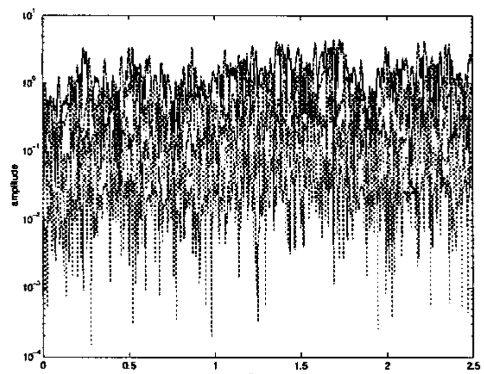

Fig. 5. A specific realization of random process defined by five Rayleigh fading channel taps ($\alpha = 0$).

We initialized the adaptive filter taps with the Wiener solution, ran the filters in training mode for 1000 BPSK symbols, and tracked the channel over 10 coherence times. Due to the large variation in the taps, the filters have to adapt significantly to keep up with the channel and avoid errors from the noisy filter taps and noisy environment. We see in Figs. 7-10 that as the channel statistics vary faster, LMS and RLS perform increasingly better for filters that can adapt more quickly, up to some optimal point. As the filters adapt more quickly, the lag between the filters taps and the optimal taps lessen, but the filter taps become quite noisy as they depend so heavily on the most recent symbol. For very slow filters, the lag of the filter weights causes significant errors, and for very fast filters, the lag error is negligible, but the taps only noisily approximate the optimal filters, resulting in many errors. The optimum point balances the error due to lag and to noisy tap updates.

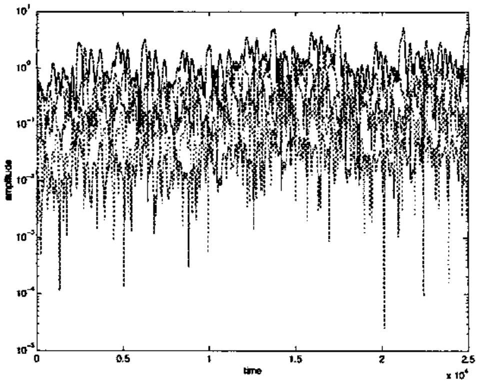

Fig. 6. A specific realization of random process defined by five Rayleigh fading channel taps ($\alpha = 1$).

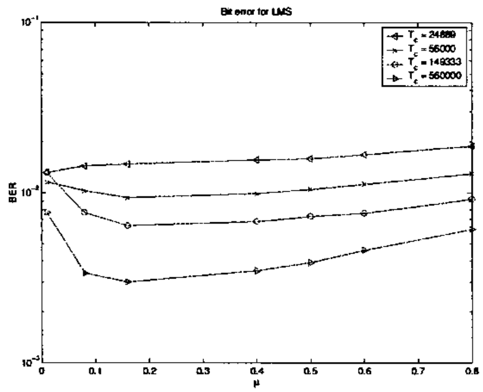

Fig. 7. Average BER vs. μ ($\alpha = 0$).

We see in Figs. 11-12 that both LMS and RLS perform better in channels with higher T_c. For smaller T_c, each algorithm needs to adapt faster at the expense of misadjustment. For larger T_c, RLS outperforms LMS noticeably, and the performance gap closes somewhat as T_c is decreased, agreeing with intuition. We found that faster filters are generally required to track faster channels (with respect to rate of change of statistics) and expected to find that LMS could outperform RLS for very quickly changing channels since RLS is required to update an entire matrix of parameters whereas LMS only has to update the vector taps. With fast changing statistics, we previously assumed RLS could not accurately update the autocorrelation matrix, but it was seen, for small values of T_c, that

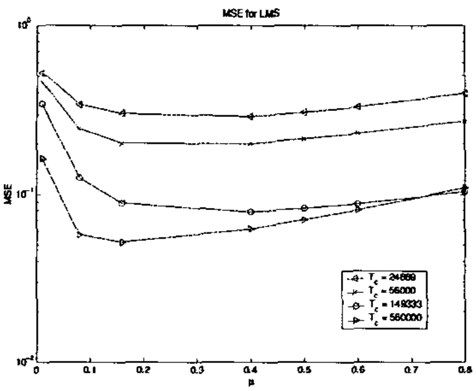

Fig. 8. MSE vs. μ ($\alpha = 0$).

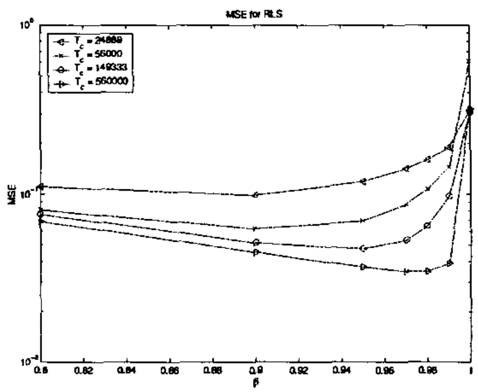

Fig. 10. MSE vs. β ($\alpha = 0$).

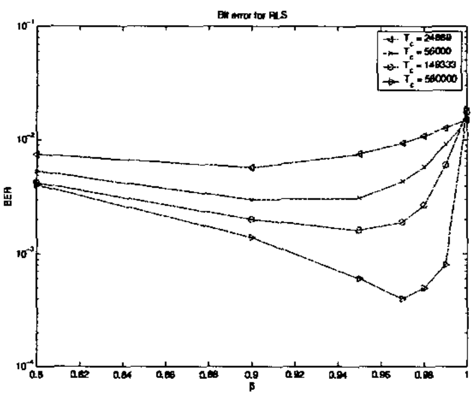

Fig. 9. Average BER vs. β ($\alpha = 0$).

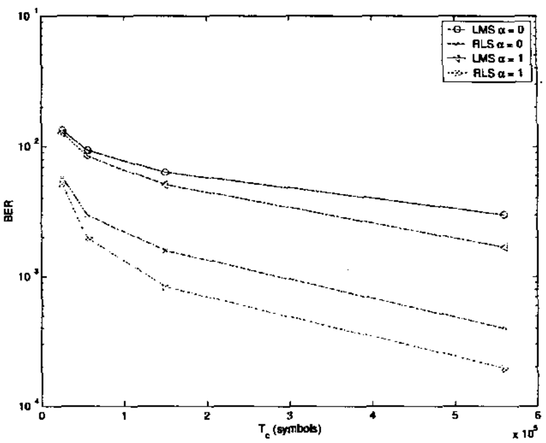

Fig. 11. BER vs. T_c.

neither algorithm effectively tracked the nonstationary channel, and they both essentially "gave up," thus LMS never outperformed RLS. We also found the performance gap between LMS and RLS to decrease as filter size increases. We postulate this is due to the same effect that we previously thought would lead LMS to outperform RLS, that RLS has to update a matrix with number of entries equal to the square of the number of filter taps so the complexity of keeping an accurate autocorrelation matrix increases as the square of the number of taps.

In Figs. 11-12, we observe that now BER (as well as MSE) is dependent on α whereas, for the no ISI case, BER is independent of α and T_c. As expected, the $\alpha = 1$, less nonstationary environment resulted in lower BER. The relationship between MSE and BER is clear from these results, in which it appears they are strongly correlated. Because almost all analytical results on adaptive filtering in the literature are in terms of MSE and not BER, being able to correlate the two metrics helps us apply much of the intuition available in current literature on adaptive filtering, providing much insight in comparing the two algorithms.

It has been proven that for stationary channels, RLS will always outperform LMS [1]. For realistic nonstationary channels, such as those of the mobile wireless ad hoc network, we have shown RLS outperforms LMS noticeably and by greater margin as T_c increases. However, because the gains are not enormous, LMS might still be preferred in some applications because of its simplicity. We note that results were found to be similar with and without lognormal shadowing, the latter scenario assuming T_c is defined as the small-scale coherence time. Thus, the same conclusions are reached for both the time-varying channel and the more specific nonstationary channel, showing that the more highly nonstationary environment induced by the latter doesn't have much of an impact on results. For nonstationary channels with large coherence times, RLS outperforms LMS since RLS is a more accurate and intricate algorithm. One would think that LMS should outperform RLS for small coherence times since RLS is so complex that it cannot keep up with accurately updating its matrix of parameters while LMS can easily update its vector taps. However, with our unique assumptions and simulation parameters, we found the RLS algorithm to always outperform the LMS algorithm, until the channel changes so fast it is better off not adapting at all.

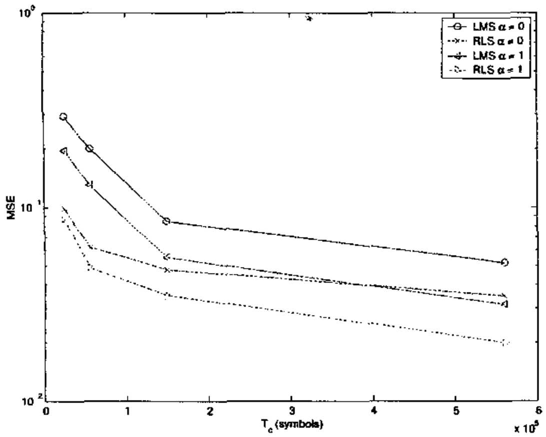

Fig. 12. MSE vs. T_c.

IV. Acknowledgment

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