# Finite Rate Feedback MIMO Broadcast Channels

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***Abstract*— This paper summarizes recent results on the multiple antenna broadcast (downlink) channel with finite rate feedback of channel state information from each receiver. In this model, the receivers are assumed to have perfect channel knowledge, while the transmitter gains channel knowledge through a finite rate feedback channel from each receiver. The SNR degradation as a function of the feedback rate is computed, as is the feedback rate scaling required to achieve throughput close to that with perfect CSIT. Additionally, it is shown that a small number of antennas per receiver can be used to significantly reduce this required feedback load. Finally, the potential multiuser diversity in finite-rate feedback systems with a large number of users is studied.**

## I. Introduction

In multiple antenna broadcast (downlink) channels, capacity can be tremendously increased by adding antennas at only the access point (transmitter) [1][2]. In essence, an access point (AP) equipped with $M$ antennas can support downlink rates up to a factor of $M$ times larger than a single antenna access point, even when the mobile devices have only single antennas However, the transmitter must have accurate channel state information (CSI) in order to realize these multiplexing gains. In frequency-division duplexed systems, training can be used to obtain channel knowledge at each of the mobile devices (receivers), but obtaining CSI at the access point generally requires feedback from each mobile.

In the practically motivated *finite rate feedback* model, each mobile feeds back a finite number of bits regarding its channel instantiation at the beginning of each block or frame. This model was first considered for point-to-point MIMO channels in [3][4][5]. In point-to-point channels, the transmitter uses such feedback to more accurately direct its transmitted energy towards the receiver, and even a small number of bits per antenna can be quite beneficial [6]. This is somewhat intuitive, because the level of CSI available at the transmitter (denoted CSIT) does not affect the multiplexing gain of point-to-point MIMO systems [7]. However, CSIT is more critical in downlink channels because it does affect the multiplexing gain [1][2][8], and thus the required feedback rate is generally larger.

We study downlink systems in which the simple transmission technique of zero-forcing beamforming is used. While zero-forcing can create independent channels to up to $M$ users (assuming there are $M$ transmit antennas), under the assumption of perfect CSIT, this is no longer possible when the transmitter has imperfect CSI, as is the case in the finite rate feedback model. In essence, imperfection in CSIT leads to multi-user interference at each receiver, which in turn reduces SINR's and throughput. This performance degradation is quantified when each of the mobiles has a single receive antenna, and it is shown that the feedback rate must be scaled approximately linearly with both the number of transmit antennas and the system SNR to achieve throughput close to perfect CSIT zero-forcing [9] (closely related results are given in [10]). However, the resulting feedback requirements can be quite large in even moderate sized systems. Thus motivated, we show that a small number of antennas at each mobile can be used to improve CSIT quality and thereby reduce the required feedback rate [11]. Finally, we summarize the results of [12]. in which systems with many users are considered, and it is shown that the transmitter must additionally be provided with information on the magnitude of the quantization error in order to realize multi-user diversity.

Notation: We use lower-case boldface to denote vectors, upper-case boldface for matrices, and the symbol $(\cdot)^H$ for the conjugate transpose. The norm of vector $\mathbf{x}$ is denoted $||\mathbf{x}||$.

## II. System Model

We consider a $K$ receiver multiple antenna broadcast channel in which the transmitter (access point or AP) has $M$ antennas, and each of the receivers has a single receive antenna[1]. The received signal at the $i$-th mobile is given by:

$$y_i = \mathbf{h}_i^H \mathbf{x} + n_i, \quad i = 1, \ldots, K \tag{1}$$

where $\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K$ are the channel vectors (with $\mathbf{h}_i \in \mathbb{C}^{M\times 1}$), the vector $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is the transmitted signal, and $n_1, \ldots, n_K$ are independent complex Gaussian noise terms with unit variance. There is a transmit power constraint of $P$, i.e., the input must satisfy $E[||\mathbf{x}||^2] \leq P$. In Sections III and IV we assume the number of mobiles is equal to the number of antennas, i.e., $K = M$. Note that randomly selecting $M$ users from a pool of more than $M$ users is equivalent to $K = M$. In Section V, however, we explicitly consider systems with $K > M$ and consider the problem of selecting users.

We consider a block fading channel, with independent Rayleigh fading from block to block (i.e., the components of the channel vectors are iid unit variance complex Gaussian).

[1] We consider multiple receive antennas in Section IV, but the proposed method simplifies the channel to a single receive antenna downlink channel equivalent to that described here.

Each of the receivers is assumed to have perfect and instantaneous knowledge of its own channel $\mathbf{h}_i$. Notice it is not required for mobiles to know the channel of other mobiles.

### A. Finite Rate Feedback Model

At the beginning of each block, each receiver quantizes its channel (with $\mathbf{h}_i$ assumed to be known perfectly at the $i$-th receiver) to $B$ bits and feeds back the bits perfectly and instantaneously to the access point. Vector quantization is performed using a codebook $\mathcal{C}$ that consists of $2^B$ $M$-dimensional unit norm vectors $\mathcal{C} \triangleq \{\mathbf{w}_1, \ldots, \mathbf{w}_{2^B}\}$. Each receiver quantizes its channel vector to the beamforming vector that forms the minimum angle to it, or equivalently that maximizes the inner product [4] [5]. Thus, user $i$ quantizes its channel to $\hat{\mathbf{h}}_i$, chosen according to:

$$\hat{\mathbf{h}}_i = \arg \max_{\mathbf{w}=\mathbf{w}_1,\ldots,\mathbf{w}_{2^B}} |\mathbf{h}_i^H \mathbf{w}| \quad (2)$$

$$= \arg \min_{\mathbf{w}=\mathbf{w}_1,\ldots,\mathbf{w}_{2^B}} \sin^2\left(\angle(\mathbf{h}_i, \mathbf{w})\right). \quad (3)$$

and feeds the index of the quantization back to the transmitter. It is important to notice that only the direction of the channel vector is quantized, and no magnitude information is conveyed to the transmitter.

In this work we use *random vector quantization* (RVQ), in which each of the $2^B$ quantization vectors is independently chosen from the isotropic distribution on the $M$-dimensional unit sphere [13]. To simplify analysis, each receiver is assumed to use a different and independently generated codebook, and we analyze performance averaged over the distribution of random codebooks.

### B. Zero-Forcing Beamforming

After receiving the quantization indices from each of the mobiles, the AP can use zero-forcing beamforming (ZFBF) to transmit data to the $M$ users. Since the transmitter does not have perfect CSI, ZFBF must be performed based on the quantizations instead of the channel realizations. When ZFBF is used, the transmit vector is defined as $\mathbf{x} = \sum_{i=1}^{M} x_i \mathbf{v}_i$, where each $x_i$ is a scalar (chosen complex Gaussian with power $P/M$) intended for the $i$-th receiver, and $\mathbf{v}_i \in \mathcal{C}^M$ is the beamforming vector for the $i$-th receiver. The beamforming vectors $\mathbf{v}_1, \ldots, \mathbf{v}_M$ are chosen as the normalized rows of the matrix $[\hat{\mathbf{h}}_i \cdots \hat{\mathbf{h}}_M]^{-1}$, and thus they satisfy $||\mathbf{v}_i|| = 1$ for all $i$ and $\hat{\mathbf{h}}_i^H \mathbf{v}_j = 0$ for all $j \neq i$. No interference cancellation is performed, and the resulting SINR at the $i$-th receiver is:

$$SINR_i = \frac{\frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_i|^2}{1 + \sum_{j\neq i} \frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_j|^2}. \quad (4)$$

Note that the interference terms in the denominator are strictly positive because $\mathbf{h}_i \neq \hat{\mathbf{h}}_i$, i.e., due to the quantization error. The long-term average rate achieved is the expectation of $\log(1 + SINR_i)$ over the distribution of the fading and RVQ.

## III. Throughput Analysis

In this section we summarize results from [9] on the sum rate performance of single receive antenna downlink channels with finite rate feedback, both for a fixed number of feedback bits as well as for an increasing (with SNR) amount of feedback. In this section we assume the number of receivers is equal to the number of transmit antennas, i.e. $K = M$. We are generally interested in comparing the long-term average rates achieved with finite rate feedback to those achieved with perfect CSIT zero-forcing beamforming.

If the transmitter has perfect CSIT, the beamforming vectors (denoted $\mathbf{v}_{ZF,i}$) can be chosen perfectly orthogonal to all other channels, thereby eliminating all multi-user interference. Thus, the SNR of each user is as in (4), but with zero interference terms in the denominator. The resulting average rate (per mobile) is:

$$R_{ZF}(P) = E_{\mathbf{H}}\left[\log\left(1 + \frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_{ZF,i}|^2\right)\right]. \quad (5)$$

Since the beamforming vector $\mathbf{v}_{ZF,i}$ is chosen orthogonal to the $(M-1)$ other channel vectors $\{\mathbf{h}_j\}_{j\neq i}$, each of which is an iid isotropic vector, the beamforming vector is also an isotropic vector, *independent* of the channel vector $\mathbf{h}_i$.

With limited feedback, ZFBF is performed based on the quantization vectors, and thus multi-user interference cannot be completely eliminated. The resulting average rate is:

$$R_{FB}(P) = E_{\mathbf{H},W}\left[\log\left(1 + \frac{\frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_i|^2}{1 + \sum_{j\neq i} \frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_j|^2}\right)\right]. \quad (6)$$

Due to the isotropic nature of the channel realizations and the quantization vectors, the channel quantization vectors $\hat{\mathbf{h}}_1, \ldots, \hat{\mathbf{h}}_M$ are also independent isotropic vectors. As a result, the beamforming vector $\mathbf{v}_i$ is isotropically distributed and is independent of the corresponding channel $\mathbf{h}_i$, as it is with perfect CSIT. Thus, the denominator in the SINR expression in (6) is the only difference between the rates achieved with perfect CSIT and with limited feedback. This observation motivates the following theorem, which approximates the SNR degradation due to finite rate feedback:

*Theorem 1:* Finite rate feedback with random vector quantization incurs an SNR degradation of approximately

$$\Delta SNR_{dB} \approx 10 \log_{10}\left(1 + P \cdot 2^{-\frac{B}{M-1}}\right)$$

where $B$ is the number of feedback bits per mobile.

This approximation is arrived at by utilizing statistics of RVQ [14] to upper bound the expectation of the denominator of the SINR with feedback:

$$E_{\mathbf{H},W}\left[1 + \sum_{j\neq i} \frac{P}{M}|\mathbf{h}_i^H \mathbf{v}_j|^2\right] \leq 1 + P \cdot 2^{-\frac{N_{FB}}{M-1}}.$$

The most important feature to notice is that the SNR loss is an increasing function of the system SNR as well as the number of AP antennas. As a result of this, a system with a

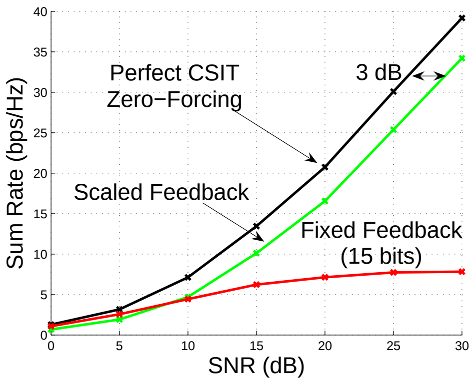


Fig. 1. Downlink Channel with $M = K = 6$, $N = 1$

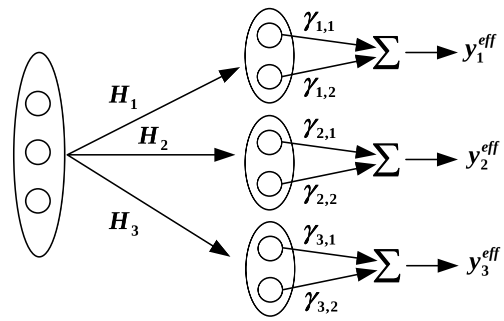

Fig. 2. Effective Channel for $M = K = 3$, $N = 2$ System

fixed number of feedback bits per mobile ($B$) will become interference-limited at high SNR. This is intuitive because the residual quantization error causes both the multi-user interference to grow linearly with the transmit power $P$, and thus leads to a bounded SINR.

In order to prevent this interference-limited behavior, the feedback quality must improve as the SNR is increased. It is desirable to achieve the full multiplexing gain (i.e., a rate curve with the same slope as perfect CSIT) as well as achieve rates that are measurably close to those achieved with perfect CSIT. We define the rate gap $\Delta(P)$ between perfect CSIT and limited feedback as:

$$\Delta(P) \triangleq R_{ZF}(P) - R_{FB}(P). \tag{7}$$

The following theorem quantifies the scaling of feedback needed to keep the rate gap $\Delta(P)$ bounded by an arbitrary constant $r > 0$ at all SNR's. Note that also ensures the full multiplexing gain is achieved.

*Theorem 2:* A rate gap $\Delta(P)$ no larger than a constant $r > 0$ is maintained at all SNR's by scaling the number of feedback bits per mobile $B$ according to:

$$B \approx \frac{M-1}{3} P_{dB} - (M-1)\log_2(2^r - 1). \tag{8}$$

This result is proven by setting the upper bound to the SNR degradation in Theorem 1, which can be shown to be an upper bound to $\Delta(P)$, equal to $r$ and solving for $B$ as a function of the SNR. Note that the feedback rate must be scaled approximately linearly with the number of transmit antennas $M$ as well as the system SNR. Note that the feedback need only be scaled linearly with the number of antennas in point-to-point MIMO systems in order to maintain a constant rate gap [13].

Since the per user rate $R_{ZF}(P)$ has a slope of 1 bps/Hz/3 dB at asymptotically high SNR (due to the multiplexing gain of $M$ in the system), a rate offset of $r$ bps/Hz corresponds to a $3r$ dB shift of either the per user rate curve or the total throughput curve. Thus, $r = 1$ corresponds to a 3 dB offset, and the resulting scaling takes on a particularly simple form:

$$B = \frac{M-1}{3} P_{dB} \quad \text{bits/mobile} \tag{9}$$

when a 3 dB gap is desired. In Fig. 1, achievable rates vs. SNR are shown for a 6 antenna, 6 user system, for perfect CSIT zero-forcing, finite rate feedback with 15 bits per mobile, and finite rate feedback with rate scaled according to (9). Note that the fixed feedback curve is bounded, while the rates achieved with scaled feedback perform within 3 dB of perfect CSI zero-forcing.

Note that regularized zero-forcing, in which the beamforming vectors are chosen to be the normalized rows of of the matrix $(\frac{M}{P}\mathbf{I} + [\hat{\mathbf{h}}_i \cdots \hat{\mathbf{h}}_M])^{-1}$, can be used instead of standard zero-forcing. Though ZF and regularized ZF are equivalent at high SNR, there can be a considerable advantage to using regularization at low and moderate SNR's [15]. Due to the high SNR equivalence, note that Theorem 2 applies to regularized ZF at asymptotically high SNR[16].

## IV. Reduced Feedback with Multiple Mobile Antennas

In this section we describe a method that utilizes $N > 1$ antennas at each mobile to reduce the quantization error, and therefore reduces the required feedback load per mobile [11]. Each mobile linearly combines its $N$ antenna outputs, thereby creating a single antenna output denoted $y_i^{\text{eff}}$. Furthermore, only the effective channel output is used when receiving transmissions, and thus the channel is eq1uivalent to a single antenna downlink channel. A 3 user channel with $M = 3$ and $N = 2$ is shown in Fig. 2. The advantage of such a system comes from intelligent selection of the linear combiner coefficients, which are chosen to yield an effective channel that can be quantized with minimal error. Note that the multiple receive antennas are only used to reduce quantization error, but effectively only a single antenna is used for reception. Thus, standard ZFBF can be used, and the transmitter need not even be aware of the number of antennas per mobile.

The $N$-dimensional received vector at the $i$-th receiver is described by $\mathbf{y}_i = \mathbf{H}_i^H \mathbf{x} + \mathbf{n}_i$, where the $M \times N$ matrix $\mathbf{H}_i$ and the noise vector $\mathbf{n}_i$ have iid unit variance complex Gaussian components. For simplicity of exposition, we focus on the received channel at user 1. Denote the $N$ columns of $\mathbf{H}_1$ by $\mathbf{g}_1, \ldots, \mathbf{g}_N$ (each in $\mathbb{C}^{M \times 1}$), i.e., $\mathbf{H}_1 = [\mathbf{g}_1 \cdots \mathbf{g}_N]$. Thus $\mathbf{g}_i$ describes the vector channel to mobile 1's $i$-th antenna.

### A. Effective Channel Quantization

We now describe the quantization procedure performed at each mobile. Consider the linear combination of the $N$-dimensional received signals by weights $\boldsymbol{\gamma}_1 = (\gamma_{1,1}, \ldots, \gamma_{1,N})$ satisfying $|\boldsymbol{\gamma}_1| = 1$:

$$\begin{aligned} y_1^{\text{eff}} \triangleq \boldsymbol{\gamma}_1^H \mathbf{y}_i &= \sum_{k=1}^{N} \gamma_{1,k}^H (\mathbf{g}_k^H \mathbf{x} + n_k) \\ &= (\mathbf{h}_1^{\text{eff}})^H \mathbf{x} + n, \end{aligned}$$

where $\mathbf{h}_1^{\text{eff}} = \sum_{k=1}^{N} \gamma_{1,k} \mathbf{g}_k = \mathbf{H}_1 \boldsymbol{\gamma}_1$ and $n = \sum_{k=1}^{N} \gamma_{1,k}^H n_k$ is unit variance complex Gaussian noise because $|\boldsymbol{\gamma}_1| = 1$. Since any set of weights satisfying the unit norm can be chosen, $\mathbf{h}_1^{\text{eff}}$ can be in *any* direction in the subspace spanned by $\mathbf{g}_1, \ldots, \mathbf{g}_N$. Quantization error is minimized by choosing $\mathbf{h}_1^{\text{eff}}$ to be in the direction that can be quantized best, or equivalently the direction which is closest to one of the quantization vectors. The corresponding channel quantization vector is the vector that forms the minimum angle with span$(\mathbf{g}_1, \ldots, \mathbf{g}_N)$:

$$\hat{\mathbf{h}}_1 = \arg \min_{\mathbf{w}=\mathbf{w}_1, \ldots, \mathbf{w}_{2^B}} |\angle(\mathbf{w}, \text{span}(\mathbf{g}_1, \ldots, \mathbf{g}_N))| \quad (10)$$

Denote the normalized projection of $\hat{\mathbf{h}}_1$ onto span$(\mathbf{g}_1, \ldots, \mathbf{g}_N)$ by the vector $\mathbf{s}_1^{\text{proj}}$. The corresponding weighting vector $\boldsymbol{\gamma}_1$, which gives $\mathbf{h}_1^{\text{eff}} = \mathbf{H}_1 \boldsymbol{\gamma}_1$ in the direction of $\mathbf{s}_1^{\text{proj}}$, is found using the pseudo-inverse of $\mathbf{H}_1$:

$$\boldsymbol{\gamma}_1 = \frac{\left(\mathbf{H}_1^H \mathbf{H}_1\right)^{-1} \mathbf{H}_1^H \mathbf{s}_1^{\text{proj}}}{|| \left(\mathbf{H}_1^H \mathbf{H}_1\right)^{-1} \mathbf{H}_1^H \mathbf{s}_1^{\text{proj}} ||}. \quad (11)$$

Each mobile computes its channel quantization and linear combination weights according to this procedure, and feeds back the quantization index to the transmitter. The weighting vector is then used to linearly combines the $N$ received signals to yield a scalar output $y_i^{\text{eff}} = (\mathbf{h}_i^{\text{eff}})^H \mathbf{x} + n$ with $\mathbf{h}_i^{\text{eff}} = \mathbf{H}_i \boldsymbol{\gamma}_i$. Note that the norm of the effective channel is given by $||\mathbf{h}_i^{\text{eff}}|| = 1/|| \left(\mathbf{H}_i^H \mathbf{H}_i\right)^{-1} \mathbf{H}_i^H \mathbf{s}_i^{\text{proj}} ||$.

### B. Throughput Analysis

The effective channel quantization procedure converts the multiple transmit, multiple receive antenna downlink channel into a multiple transmit, single receive antenna downlink channel with channel vectors $\mathbf{h}_1^{\text{eff}}, \ldots, \mathbf{h}_M^{\text{eff}}$ and channel quantizations $\hat{\mathbf{h}}_i \cdots \hat{\mathbf{h}}_M$. After receiving the quantization indices, the transmitter performs ZFBF based on the channel quantization vectors $\hat{\mathbf{h}}_i \cdots \hat{\mathbf{h}}_M$.

The resultant system has many of the same properties as the single antenna channel analyzed in Section III. It can be shown that the $M$ effective channels are independent isotropic $M$-dimensional vectors, as are the $M$ channel quantizations. The primary difference is the reduced quantization error: in a single antenna system, the quantization error is the angle between the quantization vector and a one-dimensional subspace, while for $N > 1$ the error is the angle between the quantization and an $N$-dimensional subspace [11].

Similar to Theorem 1, we can derive a simple approximation for the SNR degradation relative to *single* receive antenna downlink channels with perfect CSIT, which is the same benchmark used in Theorem 1.

*Theorem 3:* Finite rate feedback with $N$ receive antennas per mobile incurs an SNR degradation of approximately

$$\Delta SNR_{dB} \approx 10 \log_{10} \left( 1 + P \cdot 2^{-\frac{B}{M-N}} \cdot \binom{M-1}{N-1}^{-\frac{1}{M-N}} \right).$$

Though the SNR loss is considerably smaller than with $N = 1$, the quantization error is still strictly positive with probability one if $N < M$, and thus the system is also interference limited at high SNR. Using the same framework as in Theorem 2, the required feedback scaling to achieve performance close to that of perfect CSIT zero-forcing can be quantified:

*Theorem 4:* A rate gap $\Delta(P)$ no larger than $r$ between zero forcing with perfect CSI and zero forcing with limited feedback can be achieved at asymptotically high SNR by scaling the feedback rate according to:

$$\begin{aligned} B &= \frac{M-N}{3} P_{dB} - (M-N) \log_2 c \quad (12) \\ &\quad -(M-N) \log_2 \left( \frac{M}{M-N+1} \right) - \log_2 \binom{M-1}{N-1}, \end{aligned}$$

where $c = 2^r \cdot e^{-(\sum_{l=M-N+1}^{M-1} \frac{1}{l})} - 1$.

The resulting feedback savings is the difference between equations (12) and (8). For a 3 dB gap, the feedback reduction relative to a single receive antenna system can be accurately approximated as:

$$\Delta_{FB}(N) \approx \frac{N-1}{3} P_{dB} + \log_2 \binom{M-1}{N-1} - (N-1) \log_2 e.$$

The sum rate of a 6 transmit antenna downlink channel is plotted in Fig. 3. The perfect CSIT zero-forcing curve is plotted along with the rates achieved using finite rate feedback with the feedback load scaled as specified in (12) for $N = 1, 2$ and 3 and a 3 dB gap. Notice that the rates achieved for different numbers of transmit antennas are nearly indistinguishable, and all three curves are approximately 3 dB shifts of the perfect CSIT curve. Here the feedback reduction at 20 dB is 7 and 12 bits, respectively, for 2 and 3 receive antennas.

## V. Exploiting Multi-User Diversity

In this section we summarize results from [12] on finite-rate feedback multiple antenna downlink channels with many users. When there are more users than transmit antennas ($K > M$), throughput can be further increased in by intelligently selecting a set of up to $M$ users and performing ZFBF on the selected set. Note that the multiple receive antenna quantization technique described in the previous section can be used in conjunction with these methods.

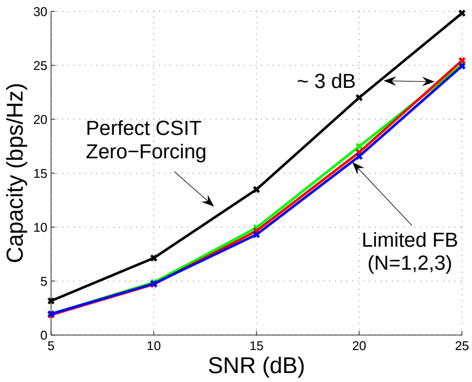


Fig. 3. Sum rate of $M = K = 6$ downlink channel

Clearly, the ideal set of users for ZFBF would have mutually orthogonal channels as well as large channel magnitudes. Although ZFBF is not capacity-achieving, ZFBF (with perfect CSIT) has been shown to asymptotically achieve the sum capacity in the limit of a large number of users [17][18]. The throughput achieved with ZFBF as well as the sum capacity grow on the order of $M \log\log K$ as the number of users is taken to infinity (while keeping $M$ and $P$ fixed), where the double-logarithmic growth is the so-called *multi-user diversity* benefit.

User selection can also be performed when the transmitter has imperfect CSIT via the feedback channel, but somewhat surprisingly this diversity benefit is not achievable [12]. Since the transmitter only has access to the channel quantizations, the ideal scenario is finding a set of $M$ users with orthogonal channel quantization vectors. The resulting ZFBF vectors would then be perfectly aligned with these quantizations (i.e., $\mathbf{v}_i = \hat{\mathbf{h}}_i$), but there is residual multi-user interference even in this ideal scenario due to the imperfect quantizations. As a result of this interference, the achievable throughput is bounded as $K \to \infty$. In fact, the same bounded behavior occurs even if the transmitter is provided with perfect knowledge of the channel magnitudes in addition to the channel quantizations, again due to lingering effects of the quantization error [12].

In order to realize multi-user diversity effects, the transmitter must also be provided information regarding the quantization error, so that, intuitively, users with small quantization error can be selected. In fact, it is sufficient for each mobile to feed back the following scalar quantity in addition to the channel quantization index:

$$\alpha_i = \frac{\frac{P}{M}||\mathbf{h}_i||^2 \cos^2\theta_i}{1 + \frac{P}{M}||\mathbf{h}_i||^2 \sin^2\theta_i}, \quad (13)$$

where $\theta_i$ is the angle between $\mathbf{h}_i$ and its quantization $\hat{\mathbf{h}}_i$ [12]. Note that $\alpha_i$ is the received SINR at the $i$-th mobile if the transmitter is able to find a set (including $\hat{\mathbf{h}}_i$) of $M$ orthogonal quantizations. Furthermore, the optimal $M \log\log K$ throughput growth can be achieved using efficient user selection algorithms based on the channel quantizations and this SINR feedback [12].

## VI. Conclusion

While tremendous capacity benefits can be gained by utilizing multiple transmit antennas in downlink channels, accurate CSIT is generally required. We have described how a finite rate feedback channel from each mobile can be used to provide the transmitter with sufficiently accurate CSI. However, the feedback requirements are generally considerably higher than in comparable point-to-point MIMO channels, even when multiple mobile antennas are used to improve the quantization accuracy. Thus, the practical viability of these techniques will depend on the availability of a relatively high rate and low latency feedback channel from mobiles to the access point, as well as on the time scaling of fading, which determines the frequency of feedback. In addition, it is necessary to extend these feedback mechanisms to wideband channels, as most current wireless systems utilize large bandwidths.

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