Blind Multiuser Detection in W-CDMA Systems with Large Delay Spread

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***Abstract*—A two-stage blind multiuser antenna array detector is proposed for the uplink of a wideband CDMA system subject to large delay spread. It is shown that it is possible to identify the symbols of all users without knowledge of their codes, channels, or directions. The key idea is that low-rank decomposition of the three-dimensional (3-D) received data array can reduce the FIR multiple-input multiple-output (FIR-MIMO) CDMA problem into standard FIR single-input multiple-output (FIR-SIMO) problems.**

***Index Terms*—Antenna arrays, code division multiaccess, matrix decomposition, signal detection.**

I. Introduction

RECENTLY, it has been shown that the problem of blind multiuser detection for the uplink of a DS-CDMA system employing a base station antenna array can be cast as a low-rank decomposition of a three-dimensional (3-D) received data array [4]. The approach allows joint recovery of all user signals with guaranteed identifiability without requiring knowledge of codes, channels, or steering vectors. The results of [4] are applicable for delay spread less than the symbol duration, i.e., in the order of several chips. In this letter, we consider wideband CDMA systems with delay spread that spans several symbols. We show that it is still possible to identify the symbols of all users (albeit under stronger conditions), and propose a blind multiuser detector that achieves symbol recovery in two stages. The first stage performs low-rank decomposition of the 3-D received data array, thereby reducing the FIR-MIMO CDMA problem into several independent FIR-SIMO problems, one per user. The second stage solves the FIR-SIMO problem for each user.

In [4], the uplink of a symbol-periodic DS-CDMA system consisting of F users with spreading gain of K chips/symbol is considered. The baseband outputs of I receive antennas are sampled at the chip rate, and data are collected for a total of J symbol periods. Let us temporarily consider a noiseless synchronous model with no delay spread. The output of the ith antenna for symbol j and chip k is

$$x_{i,j,k} = \sum_{f=1}^{F} a_{i,f} b_{j,f} c_{k,f} \tag{1}$$

for all $i = 1, \cdots, I$, $j = 1, \cdots, J$, and $k = 1, \cdots, K$, where $a_{i,f}$ is the complex attenuation coefficient between antenna i and user f, $b_{j,f}$ is the jth symbol transmitted from user f, and $c_{k,f}$ is the kth chip of the spreading code of user f. Equation (1) constitutes a rank-F decomposition of the $I \times J \times K$ 3-D (also known as three-way) received data array $\underline{\mathbf{X}}$ with typical element $x_{i,j,k}$, with each user contributing a rank-one three-way component. Define matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, with typical elements $a_{i,f}$, $b_{j,f}$, and $c_{k,f}$, respectively. Given $\underline{\mathbf{X}}$, the model in (1) is unique provided that $k_{\mathbf{A}} + k_{\mathbf{B}} + k_{\mathbf{C}} \geq 2F + 2$, where $k_{\mathbf{A}}$ is the k-rank of $\mathbf{A}$. The k-rank of $\mathbf{A}$ is the maximum number of linearly independent columns that can be drawn from $\mathbf{A}$ *in an arbitrary fashion* [2], [4]. This is the basis of the PARAllel FACtor (PARAFAC) approach in [4], which also explains how to treat the quasisynchronous case with small delay spread.

II. W-CDMA with Large Delay Spread

If the propagation environment is characterized by multipath that is local to the mobile, then multipath manifests primarily as temporal echoes (delay spread) but not spatial echoes (angle spread). If the maximum delay spread is Q symbols, then the model in (1) becomes

$$x_{i,j,k} = \sum_{(f,q)=(1,0)}^{(F,Q-1)} a_{i,f} b_{j-q,f} c_{k,f}^{(q)} \tag{2}$$

$i = 1, \cdots, I$, $j = 1, \cdots, J$, $k = 1, \cdots, K$, where $c_{k,f}^{(q)} := (s_f * h_f)(qK + k)$ is the convolution of the spreading code of user f with the respective multipath channel, evaluated at lag $\ell = qK + k$. The array response matrix associated to the model in (2) is

$$\boldsymbol{\mathcal{A}}_{I \times QF} := \left[\begin{array}{c|c|c} \begin{matrix} \mathbf{a}_1 & \cdots & \mathbf{a}_1 \\ & (Q\ times) & \end{matrix} & \cdots & \begin{matrix} \mathbf{a}_F & \cdots & \mathbf{a}_F \\ & (Q\ times) & \end{matrix} \end{array} \right] \tag{3}$$

where $\mathbf{a}_f := [a_{1,f}, \cdots, a_{I,f}]^T$. The corresponding signal and effective code matrices $\boldsymbol{\mathcal{B}}_{J \times QF}$ and $\boldsymbol{\mathcal{C}}_{K \times QF}$ are

$$\boldsymbol{\mathcal{B}} := \left[\begin{array}{c|c|c} \cdots & \begin{matrix} \begin{bmatrix} b_{1,f} \\ \vdots \\ b_{J,f} \end{bmatrix} & \cdots & \begin{bmatrix} b_{-(Q-1)+1,f} \\ \vdots \\ b_{J-(Q-1),f} \end{bmatrix} \\ & (Q\ shifts) & \end{matrix} & \cdots \end{array} \right] \tag{4}$$

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$$\boldsymbol{\mathcal{C}} := \left[\begin{array}{c|c|c} \cdots & \begin{array}{ccc} \begin{bmatrix} c_{1,f}^{(0)} \\ \vdots \\ c_{K,f}^{(0)} \end{bmatrix} & \cdots & \begin{bmatrix} c_{1,f}^{(Q-1)} \\ \vdots \\ c_{K,f}^{(Q-1)} \end{bmatrix} \\ & (Q\ columns) & \end{array} & \cdots \end{array} \right]. \quad (5)$$

Notice that $\boldsymbol{\mathcal{A}}$ consists of F buckets of Q identical columns each, hence, $k_{\boldsymbol{\mathcal{A}}} = 1$ and therefore, the basic identifiability result of [4] fails. However, ($\boldsymbol{\mathcal{A}}$, $\boldsymbol{\mathcal{B}}$, $\boldsymbol{\mathcal{C}}$) generates a model that is a special case of the so-called parallel factors with linear dependencies (PARALIND) class of models, which exhibit partial uniqueness [1]. In our present context, consider the case of $J \geq QF$ snapshots, spreading gain $K \geq QF$, and $I = 2$ antennas. Suppose that $\boldsymbol{\mathcal{A}}$ contains no zeros, and the steering vectors for different users are nonproportional. The data in (2) admit the following factorization

$$\boldsymbol{\mathcal{X}}_i = \boldsymbol{\mathcal{B}}\mathbf{D}_i\boldsymbol{\mathcal{C}}^T, \ i = 1, \cdots, I \quad (6)$$

where $\mathbf{D}_i$ is a diagonal matrix that holds the ith row of $\boldsymbol{\mathcal{A}}$ in its diagonal. Define $\mathbf{D} := \mathbf{D}_2\mathbf{D}_1^{-1}$, and absorb the first diagonal into $\boldsymbol{\mathcal{C}}^T$

$$\boldsymbol{\mathcal{X}}_1 = \boldsymbol{\mathcal{B}}\boldsymbol{\mathcal{C}}^T, \ \boldsymbol{\mathcal{X}}_2 = \boldsymbol{\mathcal{B}}\mathbf{D}\boldsymbol{\mathcal{C}}^T. \quad (7)$$

If $\boldsymbol{\mathcal{B}}$ and $\boldsymbol{\mathcal{C}}$ are full column rank (this requires persistence of excitation/independence conditions), then the singular value decomposition of the stacked data yields

$$\begin{bmatrix} \boldsymbol{\mathcal{X}}_1 \\ \boldsymbol{\mathcal{X}}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{B}} \\ \boldsymbol{\mathcal{B}}\mathbf{D} \end{bmatrix} \boldsymbol{\mathcal{C}}^T = \mathbf{U}_{2J\times QF}\mathbf{\Sigma}_{QF\times QF}\mathbf{V}^H{}_{QF\times K}. \quad (8)$$

$\boldsymbol{\mathcal{C}}$ being full column rank assures that $span(\mathbf{U}) = span([\begin{smallmatrix} \boldsymbol{\mathcal{B}} \\ \boldsymbol{\mathcal{B}}\mathbf{D} \end{smallmatrix}])$. Hence, there exists a nonsingular matrix $\mathbf{T}_{QF\times QF}$ such that

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mathcal{B}} \\ \boldsymbol{\mathcal{B}}\mathbf{D} \end{bmatrix} \mathbf{T}^T. \quad (9)$$

Construct $QF \times QF$ matrices

$$\begin{aligned} \boldsymbol{\mathcal{R}}_1 &:= \mathbf{U}_1^T\mathbf{U}_1 = \mathbf{T}\boldsymbol{\mathcal{B}}^T\boldsymbol{\mathcal{B}}\mathbf{T}^T = \boldsymbol{\mathcal{G}}\mathbf{T}^T, \\ \boldsymbol{\mathcal{R}}_2 &:= \mathbf{U}_1^T\mathbf{U}_2 = \mathbf{T}\boldsymbol{\mathcal{B}}^T\boldsymbol{\mathcal{B}}\mathbf{D}\mathbf{T}^T = \boldsymbol{\mathcal{G}}\mathbf{D}\mathbf{T}^T \end{aligned} \quad (10)$$

where $\boldsymbol{\mathcal{G}}_{QF\times QF} := \mathbf{T}\boldsymbol{\mathcal{B}}^T\boldsymbol{\mathcal{B}}$. $\boldsymbol{\mathcal{G}}$ is square nonsingular ($\boldsymbol{\mathcal{B}}$ is full column rank) and so are $\boldsymbol{\mathcal{R}}_1$ and $\boldsymbol{\mathcal{R}}_2$. It then follows from (10) that

$$(\boldsymbol{\mathcal{R}}_1\boldsymbol{\mathcal{R}}_2^{-1})\boldsymbol{\mathcal{G}} = \boldsymbol{\mathcal{G}}\mathbf{D}^{-1} \quad (11)$$

which is a standard eigenvalue problem. Due to the structure of $\boldsymbol{\mathcal{A}}$, there exist F distinct eigenvalues of multiplicity Q each. These are uniquely determined by solving the eigenvalue problem and hence, the steering vectors in $\boldsymbol{\mathcal{A}}$ are unique up to scaling. For each distinct eigenvalue λ, there exist Q corresponding eigenvectors in $\boldsymbol{\mathcal{G}}$ (by construction). Solving the eigenvalue problem, one will get Q eigenvectors for λ, which will have the same *span* as the corresponding columns of $\boldsymbol{\mathcal{G}}$. Hence, the rotational ambiguity in $\boldsymbol{\mathcal{G}}$ is confined within buckets of columns corresponding to a given eigenvalue (user). There is also freedom to permute the buckets corresponding to the different users. These ambiguities are simply carried over to the columns of $\boldsymbol{\mathcal{B}}$ and $\boldsymbol{\mathcal{C}}$, since $\mathbf{T}^T = \boldsymbol{\mathcal{G}}^{-1}\boldsymbol{\mathcal{R}}_1$, $\boldsymbol{\mathcal{B}} = \mathbf{U}_1\mathbf{T}^{-T}$, and $\boldsymbol{\mathcal{C}}^T = \boldsymbol{\mathcal{B}}^\dagger\boldsymbol{\mathcal{X}}_1$ (where $(.)^\dagger$ stands for pseudo-inverse). The following thus applies.

Result 1: Partial Uniqueness for $I = 2$: If $J \geq QF$, $K \geq QF$, $\boldsymbol{\mathcal{B}}$, and $\boldsymbol{\mathcal{C}}$ are full rank, $\boldsymbol{\mathcal{A}}$ contains no zeros, and the steering vectors for different users are nonproportional, then 1) the steering vectors in $\boldsymbol{\mathcal{A}}$ are unique up to scaling, and 2) span($[\mathbf{b}_1 \cdots \mathbf{b}_Q]$), $\cdots$, span($[\mathbf{b}_{Q(F-1)+1} \cdots \mathbf{b}_{QF}]$) [where $\mathbf{b}_l$ is the lth column of $\boldsymbol{\mathcal{B}}_{J\times QF}$ in (4)], and span($[\mathbf{c}_1 \cdots \mathbf{c}_Q]$), $\cdots$, span($[\mathbf{c}_{Q(F-1)+1} \cdots \mathbf{c}_{QF}]$) [where $\mathbf{c}_m$ is the mth column of $\boldsymbol{\mathcal{C}}_{K\times QF}$ in (5)] are unique.

Remark 1: Different delay spreads for the different users can in principle be accommodated, since all-zero columns in $\boldsymbol{\mathcal{C}}$ strike out the respective columns of $\boldsymbol{\mathcal{B}}$ and effectively reduce model order, provided that the reduced model is full rank. The delay spreads can in principle be determined by clustering eigenvalues, although this will likely be problematic in low SNR situations.

III. Least Squares Fitting

The trilinear alternating least squares (ALS) algorithm described in [4] can be modified [5] to fit (or refine eigenvalue estimates of) the model in (2), under the explicit constraint that

$$\boldsymbol{\mathcal{A}} = [\mathbf{a}_1 \quad \mathbf{a}_2 \quad \cdots \quad \mathbf{a}_F] \times \left[\begin{array}{c|c|c|c} \underline{\mathbf{1}} & \underline{\mathbf{0}} & & \underline{\mathbf{0}} \\ \underline{\mathbf{0}} & \underline{\mathbf{1}} & & \underline{\mathbf{0}} \\ \vdots & \vdots & \ddots & \vdots \\ \underline{\mathbf{0}} & \underline{\mathbf{0}} & & \underline{\mathbf{1}} \end{array} \right]$$

where $\underline{\mathbf{0}} = [0 \quad 0 \quad \cdots \quad 0]_{1\times Q}$ and $\underline{\mathbf{1}} = [1 \quad 1 \quad \cdots \quad 1]_{1\times Q}$. This serves to ensure that buckets of columns corresponding to the same steering vector are jointly recovered.

IV. Recovering Individual User Transmissions

According to our results in Section II, eigenanalysis will unravel the transmissions, and provide us with

$$\mathbf{Y}_{J\times Q}^{(f)} = \boldsymbol{\mathcal{T}}^{(f)}\mathbf{H}^{(f)} \quad (12)$$

for each user f, where $\boldsymbol{\mathcal{T}}^{(f)}$ is a $J \times Q$ Toeplitz matrix constructed out of the symbols emitted from user f [see (4)], and $\mathbf{H}^{(f)}$ is a full-rank $Q \times Q$ matrix. Due to the Toeplitz structure of $\boldsymbol{\mathcal{T}}^{(f)}$, (12) corresponds to a single-input, multiple-output (SIMO) multichannel convolution (see also [5]). This structure allows recovery of user symbols from $\mathbf{Y}_{J\times Q}^{(f)}$, provided certain relatively mild technical conditions hold, e.g., cf. [3], [6]. In particular, the columns of $\mathbf{H}^{(f)}$ can be viewed as holding the impulse response coefficients of Q equivalent FIR measurement channels of length Q samples each. Hence, our result turns the blind CDMA FIR-MIMO recovery problem into F separate FIR-SIMO problems.

V. Simulation Results

We define the sample SNR (in dB's) at the input of the multiuser receiver as SNR $= 10\log_{10}(||\underline{\mathbf{X}}||_F^2/||\underline{\mathbf{V}}||_F^2)$, where $\underline{\mathbf{X}}$ is the noise-free data, and the $||\underline{\mathbf{X}}||_F^2$ is the sum of squares of all elements of the three-way array $\underline{\mathbf{X}}$. We consider $F = 2, 4,$

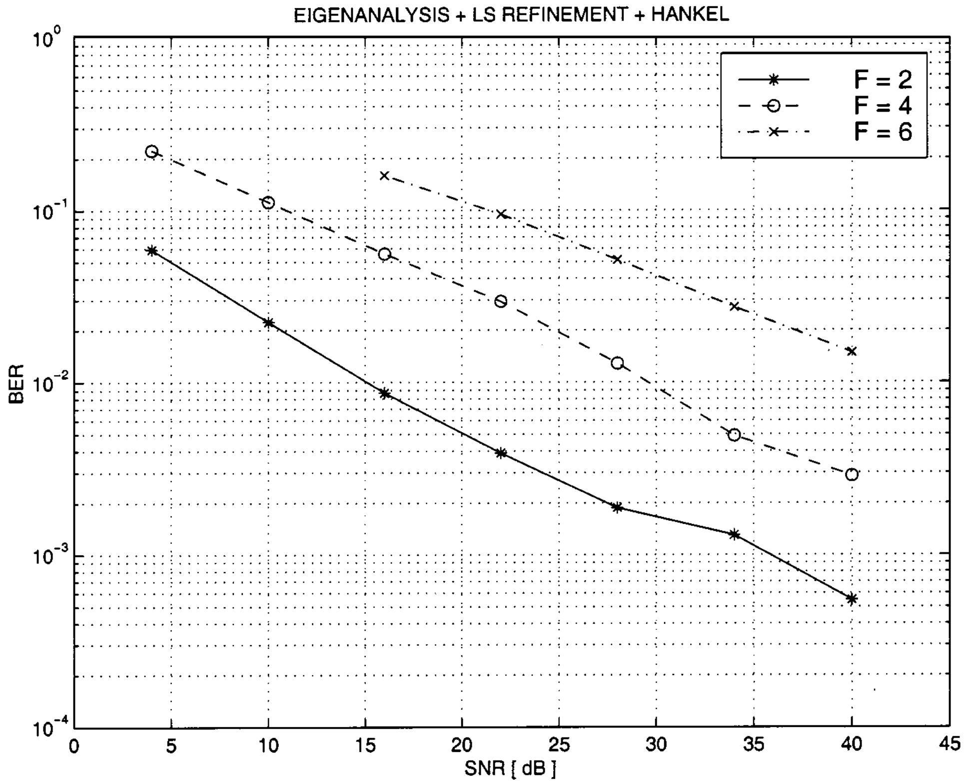

Fig. 1. BER versus SNR: average over all statistics.

and 6 BPSK user signals, spread with pseudo-random codes of length $K = 16$, multipath spanning $Q = 2$ symbol periods, $I = 2$ antennas, and $J = 50$ snapshots. For each Monte Carlo run, the multipath channel coefficients are redrawn from an i.i.d. Gaussian generator and so are the fading coefficients. User signals and spreading codes are redrawn from a pseudo-random BPSK sequence. The noise component is redrawn from an i.i.d. Gaussian generator. The results of eigenanalysis followed by least squares refinement and the Hankel kernel approach of [3] are presented in Fig. 1, which depicts average BER (all users) versus average SNR (both averaged over signal, spreading codes, multipath, fading, and Gaussian noise statistics), for 2500 MC trials [$O(10^5)$ effective averaging per datum].

VI. Conclusions

We have presented a two-step algorithm for code, channel, and direction-blind multiuser detection in antenna array W-CDMA systems with large delay spread. The algorithm builds on PARAFAC uniqueness to separate the users and turn the FIR-MIMO CDMA problem into multiple independent FIR-SIMO problems. The latter can be solved using a variety of existing techniques, with the Hankel kernel approach of [3] being one example.

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