Experimental Evaluation of Capacity Statistics for Short VDSL Loops

E. Karipidis, N. Sidiropoulos, *Senior Member, IEEE*, A. Leshem, and L. Youming

***Abstract*—We assess the capacity potential of very short very-high data-rate digital subscriber line loops using full-binder channel measurements collected by France Telecom R&D. Key statistics are provided for both uncoordinated and vectored systems employing coordinated transmitters and coordinated receivers. The vectoring benefit is evaluated under the assumption of transmit precompensation for the elimination of self-far-end crosstalk, and echo cancellation of self-near-end crosstalk. The results provide useful bounds for developers and providers alike.**

***Index Terms*—Capacity, very-high data-rate digital subscriber line (VDSL).**

I. Introduction

FIBER TO the basement (FTTB) and fiber to the curb/cabinet (FTTC) architectures have attracted considerable interest in recent years as promising low-overhead solutions for broadband network access to businesses and residential premises. Unlike, e.g., asymmetric digital subscriber line (ADSL), where the twisted-pair copper loop length is on the order of 2–3 km, FTTB/FTTC architectures entail much shorter copper segments, typically a few hundred meters. Insertion loss (IL) decays more gracefully with frequency at these lengths, potentially supporting transmission over up to 30 MHz for the shorter loops, which is considerably more than in very-high data-rate digital subscriber line (VDSL) 998. At the same time, far-end crosstalk (FEXT) becomes more prominent at these shorter loop lengths. For instance, at 75 m, FEXT behaves quite similarly to near-end crosstalk (NEXT), as is intuitive; see also [11]. For this reason, coordinated (also known as *vectored*) multiple-input multiple-output (MIMO) transmission [4], [10] becomes even more important in this context.

While there have been considerable advances in vectoring techniques, and several companies (both start-up and major players) have been developing MIMO prototypes for use in both ADSL and VDSL-like FTTB/FTTC architectures, there has been no publicly available experimental evaluation of the capacity potential of these schemes, using measured channel data. This is important for developers and providers alike, for enhanced capacity is the key selling point.

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One reason for this lack of capacity blueprints is the associated lack of broadband short-length copper channel measurements. For the NEXT-limited case, [8] provides capacity estimates based on measurements by Telcordia [7], but the data is limited to ADSL frequencies (see also [1]). As part of the EU-FP6 U-BROAD project 506790, France Telecom R&D conducted a comprehensive measurement campaign for short VDSL loops up to 30 MHz, during which IL, NEXT, and FEXT channels were measured for S88.28.4 cable of 0.4 mm gauge, comprising 14 quads, i.e., 28 loops [9]. The measured lengths were 75 m, 150 m, 300 m, and 590 m. For each length, all 378 (28 choose 2) crosstalk channels of each type (NEXT, FEXT) were measured, for a total of over 3000 crosstalk channels. For each channel, a log-frequency sweeping scheme was used to measure the in-phase/quadrature (I/Q)[1] components of the frequency response from 10 KHz–30 MHz, yielding 801 complex samples per channel. Piecewise cubic Hermite interpolation was used to convert these samples to a linear frequency scale, with $\Delta f = 4.3125$ kHz spacing. For details on the measurement process and apparatus, see [11]. This letter describes the results of the associated capacity analysis.

The capacity of copper transmission channels depends strongly on the realization of crosstalk; that is, the selection and type of active loops in the binder. Thus, capacity is a random variable, characterized by a density parameterized by the type and number of crosstalk interferers. For brevity, we report only key capacity statistics (minimum, mean, maximum capacity per loop) parameterized by loop length. We consider both "light" and "full," FEXT-only, and NEXT plus FEXT crosstalk models, as well as the effect of coordination.

In the downstream direction, one can distinguish two MIMO communication scenarios of interest, as illustrated in Fig. 1. Point-to-point (P2P), where not only the transmitters, but also the receivers, of the MIMO subsystem are physically co-located, and thus joint receive processing is possible; and point-to-multipoint (P2M), where joint receive processing is not possible. Whereas the capacity of P2P MIMO can be readily calculated under standard assumptions, computing the capacity for P2M scenarios is far more difficult. The reason is that the P2M MIMO case corresponds to a nondegraded Gaussian broadcast channel (GBC), whose capacity can only be computed via numerical optimization (see [5] and references therein). A further complication in our context is that the alien crosstalk is colored, and typically unknown. Still, the capacity of a P2M MIMO system is bounded above by the capacity of the associated P2P MIMO system. For these reasons, we focus on computing P2P capacity statistics. Similar comments hold for the upstream direction, except that it is relatively easier to compute the (so-called *medium-access control*) capacity when joint receive processing is possible, but the transmitters cannot be coordinated.

[1]Note that magnitude information alone is not sufficient for assessing capacity in the case of coordinated transmission—phase is also important.

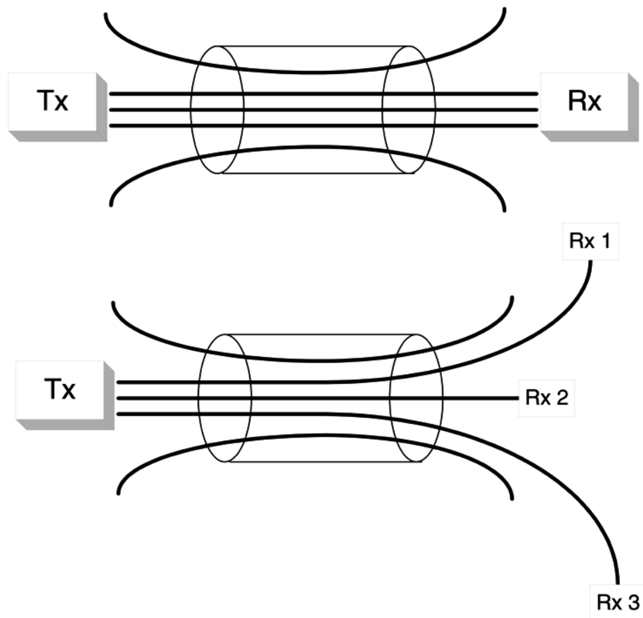

Fig. 1. Downstream MIMO scenarios. P2P (top) versus P2M (bottom).

We begin with some preliminaries (Section II), including exactly how capacity densities were evaluated. We then report our main findings in Figs. 2 and 3, which are interpreted in Section III.

II. Capacity Evaluation Method

Let $L, L_{\rm N}, L_{\rm F}$ denote the number of loops employed for coordinated transmission, and NEXT and FEXT interferers, respectively. The additive white Gaussian noise (AWGN) power spectral density (PSD) is denoted by σ^2.

For a single direct loop ($L = 1$), and a certain configuration of interfering loops, the capacity[2] is given by [2], [6]

$$C = \int_{\rm BW} \log_2(1 + {\rm SINR}(f))\,{\rm d}f \tag{1}$$

where BW denotes the available bandwidth, and the signal-to-interference-plus-noise ratio (SINR) is given by

$${\rm SINR}(f) = \frac{|H_{\rm IL}(f)|^2}{\frac{\sigma^2}{p(f)} + \sum_{i=1}^{L_{\rm N}} |H_{\rm N,i}(f)|^2 + \sum_{j=1}^{L_{\rm F}} |H_{\rm F,j}(f)|^2}. \tag{2}$$

Here, $H_{\rm IL}(f)$ is the IL (frequency response) of the direct channel, $H_{{\rm N},i}(f)$ ($H_{{\rm F},j}(f)$) is the frequency response of the ith NEXT (jth FEXT, respectively) channel, and $p(f)$ is the PSD (or, *spectral mask*) employed for the direct channel, as well as for the crosstalk channels.

In the vectored case ($L > 1$), the capacity of the coordinated MIMO subsystem (assuming joint transmit *and* joint receive processing, and multicarrier transmission with a fixed common spectral mask) is given by

$$C = \int_{\rm BW} \log_2 \det\left(\boldsymbol{I} + p(f)\boldsymbol{H}(f)\boldsymbol{R}_{nn}^{-1}(f)\boldsymbol{H}^{\rm H}(f)\right)\,{\rm d}f \tag{3}$$

[2]Throughout, we assume that the transmit PSDs of all modems are fixed to the respective regulatory spectral masks, as is common in DSL systems. In particular, PSD optimization is not considered.

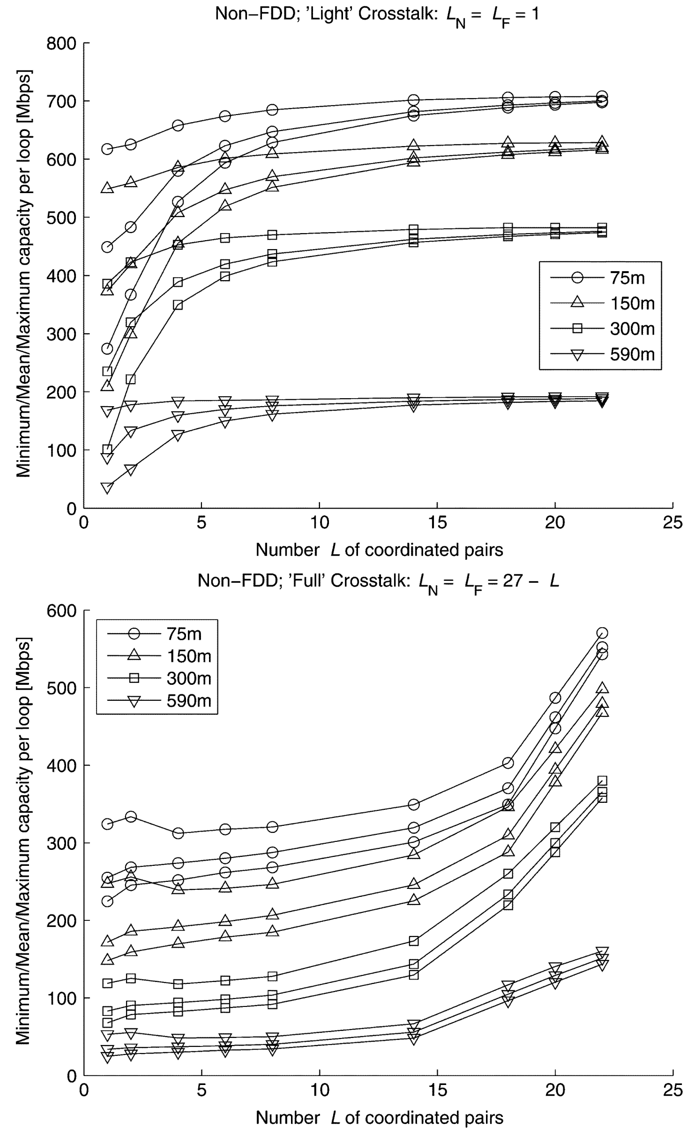

Fig. 2. Non-FDD scenario. "Light" ($L_{\rm N} = L_{\rm F} = 1$) versus "Full" ($L_{\rm N} = L_{\rm F} = 27 - L$) crosstalk.

where $\boldsymbol{H}(f)$ is the $L \times L$ input–output MIMO channel transfer matrix at frequency f, $^{\rm H}$ denotes the Hermitian (conjugate) transpose, and $\boldsymbol{R}_{nn}(f)$ is the $L \times L$ interference-plus-noise covariance matrix at the output of the MIMO subsystem at frequency f

$$\boldsymbol{R}_{nn}(f) = p(f)\boldsymbol{G}_{\rm N}(f)\boldsymbol{G}_{\rm N}^{\rm H}(f) + p(f)\boldsymbol{G}_{\rm F}(f)\boldsymbol{G}_{\rm F}^{\rm H}(f) + \sigma^2\boldsymbol{I} \tag{4}$$

where $\boldsymbol{G}_{\rm N}(f)$ is an $L \times L_{\rm N}$ crosstalk transfer matrix, whose (m, ℓ)-element is the complex coupling coefficient from the ℓth NEXT disturber to the mth loop in the vectored subsystem at frequency f; and similarly for the $L \times L_{\rm F}$ FEXT coupling matrix, $\boldsymbol{G}_{\rm F}(f)$.

For the loop lengths considered here (up to 590 m), IL is at least 30 dB less than FEXT attenuation. The matrix $\boldsymbol{H}(f)$ is therefore diagonally dominated, and it is possible to pre-equalize it at the transmitter's side without a significant

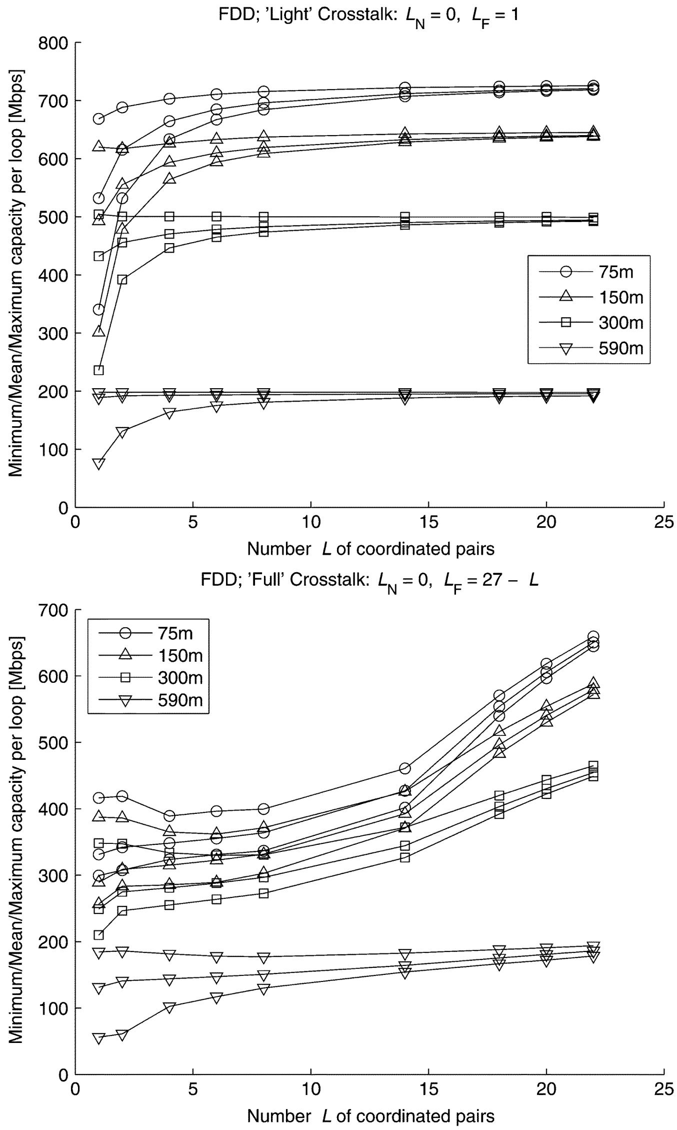

Fig. 3. FDD scenario. "Light" ($L_{\mathrm{N}} = 0, L_{\mathrm{F}} = 1$) versus "Full" ($L_{\mathrm{N}} = 0, L_{\mathrm{F}} = 27 - L$) crosstalk.

penalty[3] in terms of transmission power [3]. Thus, upon premultiplication by

$$\boldsymbol{H}^{-1}(f)\,\mathrm{diag}([H_{\mathrm{IL},1}(f),\ldots,H_{\mathrm{IL},L}(f)]) \quad (5)$$

the effective MIMO channel-transfer matrix becomes

$$\boldsymbol{H}_{\mathrm{eff}}(f) = \mathrm{diag}([H_{\mathrm{IL},1}(f),\ldots,H_{\mathrm{IL},L}(f)]). \quad (6)$$

Note that we do not invert the direct IL channels; that would entail a significant power penalty, especially at higher frequencies. Furthermore, we assume that all IL channels of the vectored subsystem are approximately equal. This is well-justified, for IL primarily depends on length, termination, and bridge taps. Then, the effective MIMO channel-transfer matrix becomes $\boldsymbol{H}_{\mathrm{eff}}(f) = H_{\mathrm{IL}}(f)\boldsymbol{I}$, and thus (3) further simplifies to

$$C = \int_{\mathrm{BW}} \log_2 \det\left(\boldsymbol{I} + |H_{\mathrm{IL}}(f)|^2 p(f)\boldsymbol{R}_{nn}^{-1}(f)\right) \mathrm{d}f. \quad (7)$$

[3]Under 3% for our data. Also note that the assumption of joint receive processing implies that all loops have equal length, which means that the off-diagonal elements of $\boldsymbol{H}(f)$ are of the same order.

This equation yields the capacity of the MIMO subsystem when self-FEXT (from within the vectored subsystem) has been precompensated at the transmitter by the diagonalization procedure in (5) and (6). However, external (often called *alien*) FEXT is still accounted for in the $p(f)\boldsymbol{G}_{\mathrm{F}}(f)\boldsymbol{G}_{\mathrm{F}}^{\mathrm{H}}(f)$ term of the covariance matrix $\boldsymbol{R}_{nn}(f)$ in (4).

Self-NEXT at the receiver can be mitigated by employing *echo-cancellation* techniques, which essentially amount to subtracting self-NEXT interference to a given loop from other loops in the coordinated subsystem, taking into account the associated frequency-dependent coupling factor. If upstream–downstream frequency-division duplex (FDD) is further employed, then alien NEXT is effectively suppressed, as well. In this case, only alien FEXT remains. In non-FDD systems, however, alien NEXT is the performance-limiting factor, for FEXT is usually much lower than NEXT, even for relatively short loops (one significant exception is very short loops, under 100 m, where FEXT looks much like NEXT, for obvious reasons).

We consider a non-FDD, echo-cancelled, transmit precompensated coordinated subsystem of order L, limited by alien crosstalk (NEXT plus FEXT). A FEXT-limited FDD scenario is also considered. In both cases, we compute the total link capacity, which can be split between upstream and downstream using, e.g., time-division multiple access (TDMA) or a suitable band plan. We therefore set $p(f) = -60$ dBm/Hz (i.e., 10^{-9} W/Hz) across the usable bandwidth. σ^2 is set to -140 dBm/Hz (i.e., 10^{-17} W/Hz), as is typical for DSL systems.

We are interested in measuring the per-loop capacity of the coordinated subsystem; that is, the MIMO capacity C in (7) divided by L. Capacity in (7) depends on the particular configuration of coordinated loops and interferers in the binder. The clear-cut way of calculating the capacity distribution is to loop over all (28 choose L) possible configurations of the L loops comprising the vectored subsystem, and, for each such configuration, loop over all ($28 - L$ choose I) possible configurations of the I alien interferers. In the FDD scenario, $L_{\mathrm{N}} = 0$, and $L_{\mathrm{F}} = I$; while in the non-FDD scenario, $L_{\mathrm{N}} = L_{\mathrm{F}} = I$, because an active loop generates both NEXT and FEXT. For each combination, we compute a Riemann sum approximation of the integral in (7), with spacing $\Delta f = 4.3125$ KHz. This entails the evaluation of the covariance in (4) for each frequency bin. We put the measured channel data corresponding to the given configuration and frequency bin into (4), compute the inverse of the covariance matrix, and insert the result into (7). For each scenario, we compute the resulting minimum, mean, and maximum capacity.

We assume that *quads* [9] are used for vectoring when L is even. This is likely to be the case in practice, and it reduces the number of possible configurations of the vectored subsystem to (14 choose $L/2$). Still, certain combinations for the numbers of the vectored and the interfering loops generate an immense number of possible configurations; for instance, $L = 8$, and $L_{\mathrm{N}} = L_{\mathrm{F}} = 8$ generates about 126 million possibilities, for each of which the integral in (7) must be approximated. For this reason, we let L take all possible values, but restrict the choice of the number of alien interferers to two extreme cases: the case of "light" interference (only one interfering loop $I = 1$); and "full" crosstalk (all but one of the remaining loops in the binder are interfering $I = 27 - L$).

III. Findings and Discussion

The results are presented in Figs. 2 (non-FDD scenario) and 3 (FDD scenario). In both figures, the upper panel shows results for "light" crosstalk, whereas the lower panel shows the corresponding results for "full" crosstalk. In each panel, three curves (minimum, mean, and maximum capacity) are plotted as a function of L for each of the four cable lengths.

Mean per-loop capacity ranges from about 0.72 Gb/s down to 34 Mb/s, depending on length and scenario considered. The mean per-loop capacity with coordination is typically under $2\times$ the capacity without coordination, but in certain cases, it can be as high as $4.4\times$ (cf. the 590-m curves in the lower panel of Fig. 2). Note that coordination significantly reduces the capacity spread even in lightly loaded systems, which is important from the operators' perspective. For "light" crosstalk, $L = 8$ is a breakpoint, beyond which virtually all of the coordination benefits are reaped. For "full" crosstalk, one has to coordinate at least half the binder $(L = 14)$ in order to see any significant improvement. A comparative discussion of the results follows.

- FDD case (Fig. 3), "full" crosstalk (lower panel): As the number of coordinated pairs (L) increases, alien FEXT pairs become self-FEXT pairs, and self-FEXT is precancelled at the transmitter. Hence, FEXT is *removed* as L increases.
- FDD case (Fig. 3), "light" crosstalk (upper panel): There is only one FEXT for all L, and thus, mean per-loop capacity does not increase drastically with L.
- Non-FDD case (Fig. 2), "full" crosstalk (lower panel): The situation here is similar to the corresponding FDD case in Fig. 3, but, in addition to FEXT, NEXT is also removed as L increases (because of echo cancellation of self-NEXT within the vectored subsystem). For this reason, the improvements for $L > 14$ are more pronounced.
- Non-FDD case (Fig. 2), "light" crosstalk (upper panel): This case is different, due to the presence of a single dominant NEXT interferer (absent from the upper panel of Fig. 3). We see more significant improvements in mean per-loop capacity and capacity spread with increasing L, up to $L = 8$. Note that the signal dimension increases with L, while the interference covariance is essentially rank-one in this case.

The statistics provided here are relatively optimistic, in the sense that they do not account for certain practical issues, such as shaping loss due to modulation, noise margin, coding gain, etc. Nevertheless, they are useful bounds on what is attainable in practice without spectral optimization. For instance, every 3 dB in gap yields capacity loss $\sim \mathrm{BW}$ Mb/s, for high SINR.

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