Joint Hop Timing and Frequency Estimation for Collision Resolution in FH Networks

Xiangqian Liu, *Member, IEEE*, Nicholas D. Sidiropoulos, *Senior Member, IEEE*, and Ananthram Swami, *Senior Member, IEEE*

***Abstract*—With the rapid growth of frequency-hopped (FH) wireless networks, interference due to frequency collisions has become one of the main performance-limiting challenges. This paper proposes a novel multiuser detection method for joint hop timing and frequency estimation, which is capable of unraveling and demodulating multiple FH transmissions in the presence of collisions and unknown hop patterns without retransmission. The method is based on the principle of dynamic programming (DP) coupled with two-dimensional harmonic retrieval (2-D HR) or low-rank trilinear decomposition, and it remains operational even with multiple unknown hop rates, frequency offsets, and asynchronism. The model is based on frequency-shift keying (FSK) and phase-shift keying (PSK) modulation, but the algorithms are also evaluated with Gaussian minimum-shift keying (GMSK) modulation and shown to be robust.**

***Index Terms*—Collision resolution, frequency hopping, harmonic retrieval (HR), multiuser detection, timing estimation.**

I. Introduction

FREQUENCY-HOPPED spread spectrum (FHSS) has been widely studied and mainly used for military applications, such as in the single-channel ground and airborne radio system (SINCGARS), due to its low probability of detection and interception, power-control issues in a peer-to-peer setting, and its inherent robustness to near–far effects [26]. Recently, it has been adopted in various types of wireless networks, for example, in home area network (HAN) and Bluetooth personal area network (PAN). With the increasing popularity of wireless networking, multiple networks are likely to coexist in a physical environment, especially in dense business and financial districts. Without coordination among coexisting FH networks, interference due to frequency collisions can become a major performance-limiting factor [13]. Interference also comes from other legitimate users in the 2.4-GHz band, such as cordless telephones and microwave ovens. When collisions occur, the received packets need to be discarded without recovering data. Subsequent retransmissions possibly induce new collisions, hence, throughput decreases and delay can become excessive. Relying on recent advances in the theory of multidimensional harmonic and low-rank analysis, and its successful applications in signal processing [15], [21], this paper develops a new approach to jointly estimate hop timing and frequencies of multiple FH signals in the presence of collisions, without the knowledge of their hop sequences or hop rates, and thus resolve collisions without retransmission.

Partial collisions can be overcome to a certain extent, using forward error correction (FEC) and interleaving (albeit at the cost of complexity, latency, and rate). Our approach is complementary to the traditional FEC schemes in that it addresses a more challenging problem wherein FEC is not viable. In the intercept mode, the receiver does not have knowledge of hop codes (or even synchronization), and FEC (even if the FEC code is assumed to be known) is useless. If the hop code and timing of a user of interest are known, and the system is lightly loaded (collisions are rare), FEC may be sufficient. However, FEC protection has been shown to break down in moderately loaded systems [28], whereas the proposed approach can resolve full collisions.

A rotational-invariance approach to collision resolution has been proposed in [31] for random access networks. The approach relies on simultaneous retransmission of collided packets. Hence, it is impossible to implement across uncoordinated FH networks. Several papers have been published on the subject of (joint) multiuser detection for FH systems, e.g., [6] and [17]. These assume, among other things, that the hop patterns of all users are known to the receiver, hence clearly are not applicable in the context of multiple noncooperative FH networks where hop patterns and timing of interfering users are unknown. An algorithm is proposed in [30] for blind estimation of the hop pattern of a single user, treating the remaining users as white Gaussian interference. This technique is conceptually simple, but stakes no identifiability claims. In addition, the method in [30] assumes perfect channel knowledge and only addresses the case of slow FH systems.

Without assuming knowledge of the hop patterns, several methods have been proposed for blind or semi-blind hop timing and frequency estimation. For example, assuming known hop rate, channelized receivers have been proposed for semi-blind hop-timing estimation (knowledge of frequency channelization is required), e.g., in [2] and [22]. However, the performance of those receivers degrades rapidly if the channelization is imperfect, or users have different hop rates. In [16], a two-step

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X. Liu is with the Department of Electrical and Computer Engineering, University of Louisville, Louisville, KY 40292 USA (e-mail: x.liu@louisville.edu).

N. D. Sidiropoulos is with the Department of Electronic and Computer Engineering, Technical University of Crete, 73100 Chania—Crete, Greece (e-mail: nikos@telecom.tuc.gr).

A. Swami is with the Army Research Lab, AMSRD-ARL-CI-CN, Adelphi, MD 20783 USA (e-mail: a.swami@ieee.org).

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scheme was developed for blind direction-of-arrival (DOA), hop timing, and frequency estimation of multiple FH signals. It does not rely on channelization and hence, is robust to frequency offset. However, it involves first detecting a hop-free data subset, and achieves single-user tracking after multiuser separation through beamforming. Detecting such a hop-free subset is difficult and unreliable at high hop rates, in the presence of collisions, under low signal-to-noise ratio (SNR), or combinations thereof. For this reason, it is also of interest to directly solve the joint multiuser hop-timing detection and carrier-estimation problem.

The multiuser detection method proposed in this paper is based on the principle of dynamic programming (DP) coupled with two-dimensional harmonic retrieval (2-D HR). If a hop-free dataset were available, one could model the signal as a mixture of (modulated) complex harmonics. Cast in matrix form, such a signal has a Vandermonde structure in the time domain. In addition, the use of a uniform linear array (ULA) induces the Vandermonde structure in the spatial domain (assuming no mutual coupling of the antenna elements). Joint DOA and hop-frequency estimation has been proposed before, e.g., in [11] and [29]. Here, we exploit the Vandermonde structure as well; more importantly, we use a 2-D HR algorithm that draws upon the rich identifiability and near-optimality results in [14], and without assuming a hop-free subset of data. We develop a DP algorithm to implement a joint maximum-likelihood (ML) estimator that yields estimates of hop timing as well. Each user may have different hop timing and rate, and frequencies may be chosen from different candidate sets.

The paper is organized as follows. We formulate the problem in Section II. In Section III, we describe the joint timing-and-frequency estimation method, i.e., DP and 2-D HR (DP-2DHR). Extensions to the cases of multipath and 2-D antenna arrays are treated in Section IV. Simulation results are given in Section V. Some notational conventions used in this paper: $\mathbf{A}^{\mathrm{T}}$, $\mathbf{A}^*$, $\mathbf{A}^{\mathrm{H}}$, and $\mathbf{A}^\dagger$ are the transpose, complex conjugate, conjugate transpose, and pseudoinverse of matrix $\mathbf{A}$; $\mathbf{A} \odot \mathbf{B}$ is the Khatri–Rao (columnwise Kronecker) product of $\mathbf{A} \in \mathbb{C}^{I\times F}$ and $\mathbf{B} \in \mathbb{C}^{J\times F}$; and $\|\cdot\|_{\mathrm{F}}$ is the Frobenius norm.

II. Problem Formulation

The scenario under consideration is shown in Fig. 1. Multiple uncoordinated FH networks are closely located with overlapped basic service areas (BSAs). Each network consists of one (or more) access points (APs) and associated wireless stations. Suppose AP1 is the receiver under study, and signals from d users are received at AP1, each from a nominal DOA with negligible angle spread. Some of the users are associated with networks of AP2 and AP3 though they are within the BSA of AP1, for example, the $(d-1)$th and dth users in Fig. 1. AP1 does not have knowledge of the hop pattern and timing of these users.

The AP is assumed to be equipped with a ULA of M antennas with baseline separation Δ wavelengths. The array steering vector in response to a signal from direction α is $\mathbf{a}(\theta) = [1\ \theta\ \cdots\ \theta^{M-1}]^{\mathrm{T}}$, where $\theta = \mathrm{e}^{j2\pi\Delta\sin(\alpha)}$. The received signal is sampled at a sampling rate of $1/T$ (T is normalized to 1),

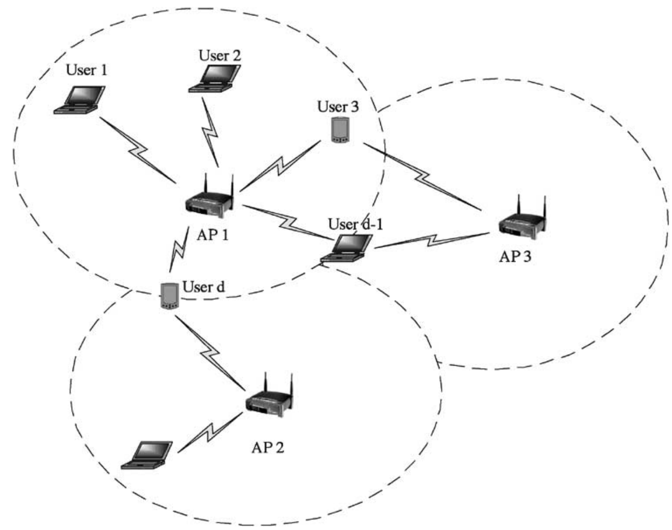


Fig. 1. Multiple coexisting FH wireless networks.

and the $M \times 1$ signal vector collected at the ULA output at sampling time n can be expressed as

$$\mathbf{x}(n) = \sum_{r=1}^{d} \mathbf{a}\left(\theta_r^{(p)}\right) \beta_r^{(p)} s_r(n) + \mathbf{w}(n) \qquad (1)$$

where $s_r(n) = \mathrm{e}^{j\omega_r^{(p)} n}$, and $\omega_r^{(p)}$ is the frequency of the transmitted signal from the rth user during its pth hop. Note that the baseline separation Δ (measured in wavelength units) is frequency dependent, hence so are the steering vectors. For notational clarity, sometimes we do not explicitly denote this dependence as long as it is clear from the context. The transmitted signals can be fast frequency hopping (FSH) or slow (SFH), with frequency-shift keying (FSK) or linear modulation [e.g., phase-shift keying (PSK) or quadratic-amplitude modulation (QAM) can also be accommodated]. $\beta_r^{(p)}$ is the complex path loss for the rth user during its pth hop that collects the (frequency-dependent) channel attenuation; the signal's initial phase $\phi_r^{(p)}$ is also absorbed into $\beta_r^{(p)}$. Here, the carrier shifts due to hopping or symbol modulation are treated as conceptually equivalent, albeit of different magnitude. $\mathbf{w}(n)$ is complex white Gaussian noise with variance σ^2. Suppose N samples (snapshots) are collected at the array output, then the received data matrix can be written as $\mathbf{X} = [\mathbf{x}(0)\ \cdots\ \mathbf{x}(N-1)]$.

Our objective is to estimate hop timing (i.e., hop instants) and hop-frequency sequences of all transmitted signals from $\mathbf{X}$, in the presence of possible collisions in some time segments, without the knowledge of users' hop patterns or hop rates; thus, collisions may be resolved and symbols may be demodulated correctly without retransmission.

In model (1), we assume a single-path transmitter–receiver propagation for each user. In Section IV, the model is generalized to incorporate multipath propagations with small delay spread, as encountered in indoor wireless environments [5], [20], [23]. It will also be shown that the algorithm developed in Section III is able to cope with the generalized multipath model. We further assume that the number of users (and, in a multipath scenario, the total number of paths for all

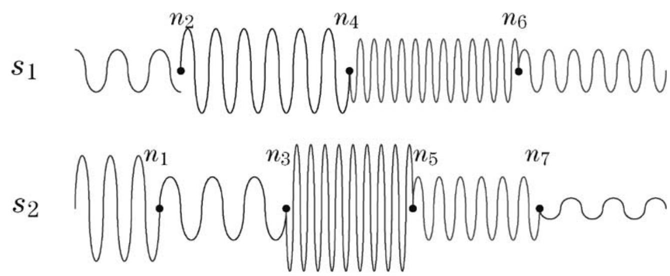


Fig. 2. FH signals transmitted from two users.

users) has already been estimated by an appropriate detection method, such as rank (see e.g., [9]) or information-theoretic criteria (e.g., Akaike Information Criterion (AIC) [1], minimum description length (MDL) [18], and MDL for coherent signals [27]). The effect of model-order mismatch will be explored in the simulations.

III. Joint Hop Timing and Frequency Estimation

For simplicity of exposition, let us focus on an FH system where there are two active users. As shown in Fig. 2, the transmitted signals s_1 and s_2 may have different hop rates and hop timing, and n_i, $i = 0, \dots, K-1$, are the hop instants ($n_0 = 0$ and $n_K = N$ by convention). We assume that within one received data block, the total number of hops for both users is bounded above by $K-1$ (such a bound could be deduced from the spectrogram of the data, and need not be tight).

Between any two systemwide consecutive hop instants, e.g., n_i and n_{i+1}, there are only two temporal frequencies involved. During such a time segment, the received data may be written as

$$\mathbf{X}_i = [\mathbf{x}(n_i) \ \cdots \ \mathbf{x}(n_{i+1}-1)] = \mathbf{A}_i \mathbf{B}_i \mathbf{S}_i^{\mathrm{T}} + \mathbf{W}_i \quad (2)$$

where $\mathbf{A}_i = [\mathbf{a}(\theta_1^{(p)}) \ \mathbf{a}(\theta_2^{(q)})]$, $\mathbf{B}_i = \mathrm{diag}(\beta_1^{(p)} \ \beta_2^{(q)})$, and the subscript $_i$ is a time index indicating that the time segment is delimited between n_i and $n_{i+1}-1$, i.e., the ith systemwide dwell. In (2), the signal matrix $\mathbf{S}_i$ is defined as

$$\mathbf{S}_i = \begin{bmatrix} \mathrm{e}^{j\omega_1^{(p)} n_i} & \mathrm{e}^{j\omega_1^{(p)}(n_i+1)} & \dots & \mathrm{e}^{j\omega_1^{(p)}(n_{i+1}-1)} \\ \mathrm{e}^{j\omega_2^{(q)} n_i} & \mathrm{e}^{j\omega_2^{(q)}(n_i+1)} & \dots & \mathrm{e}^{j\omega_2^{(q)}(n_{i+1}-1)} \end{bmatrix}^{\mathrm{T}}$$

and $\mathbf{W}_i$ is the corresponding noise matrix. Here, we assume users 1 and 2 are in their pth and qth hops, respectively, during this time segment, and $\mathbf{a}(\theta_1^{(p)})$ and $\mathbf{a}(\theta_2^{(q)})$ are the antenna steering vectors corresponding to $\omega_1^{(p)}$ and $\omega_2^{(q)}$, respectively. Since both $\mathbf{A}_i$ and $\mathbf{S}_i$ are Vandermonde matrices, the estimation of DOAs and frequencies from $\mathbf{X}_i$ in (2) is, in fact, a 2-D constant-modulus HR problem, and there are two frequency components along each of the spatial and temporal dimensions. If d users are active in the system, a similar 2-D harmonic-mixture model can be obtained except that the number of frequency components in such a time segment along each dimension is d. Recently, improved identifiability results and algorithms regarding 2-D HR have been developed [14], [15]. These are summarized as follows.

A. 2-D Harmonic Retrieval (2-D HR)

Constant-modulus 2-D HR has a wide range of applications, e.g., in sensor-array processing, wireless communications, and radar. In general terms, the 2-D constant-modulus HR problem can be stated as follows: Given a mixture of F 2-D exponentials

$$x_{g,l} = \sum_{f=1}^{F} c_f \mathrm{e}^{j\omega_f (g-1)} \mathrm{e}^{j\nu_f (l-1)} \quad (3)$$

for $g = 1, \dots, G$, and $l = 1, \dots, L$, where ω_f, $\nu_f \in \Pi$ with $\Pi := (-\pi, \pi]$, and $c_f \in \mathbb{C}$, find the parameter triples (ω_f, ν_f, c_f), for $f = 1, \dots, F$. An important problem associated with 2-D HR is the determination of the maximum number of harmonics that can be resolved from a given $G \times L$ sample in the noiseless case, which is an identifiability issue. The following statistical result is the most relaxed identifiability condition for 2-D HR to date [15].

Theorem 1: Given a sum of F 2-D undamped exponentials as in (3), for $g = 1, \dots, G \geq 3$, and $l = 1, \dots, L \geq 3$, the parameter triples (ω_f, ν_f, c_f), $f = 1, \dots, F$, are $P_{\mathcal{L}}(\Pi^{2F})$ almost surely unique, where $P_{\mathcal{L}}(\Pi^{2F})$, the distribution used to draw the $2F$ frequencies (ω_f, ν_f), $f = 1, \dots, F$, is assumed to be continuous with respect to the Lebesgue measure in Π^{2F}, provided that $F \leq \lceil G/2 \rceil \lceil L/2 \rceil$ [15].

In the context of our model in (2), Theorem 1 implies that for the given data block, if $d \leq \lceil M/2 \rceil \lceil (n_{i+1} - n_i)/2 \rceil$, then DOAs and carrier frequencies of the d users can be uniquely recovered from $\mathbf{X}_i$. Note that according to the identifiability theorem, it is possible to uniquely recover the parameters even when the number of users is much larger than the number of antenna elements. Further note that Theorem 1 demands $M \geq 3$ antennas, and at least three samples per hop-free segment, thus, at least three samples per symbol for SFH, or three samples per hop for FFH. We remark that, for a single 2-D harmonic, it is possible to recover the associated parameters even with two samples per dimension. In certain cases, it is also possible to recover the parameters of a mixture of several 2-D harmonics with only two samples along one dimension; however, proving a general identifiability result for this case is complicated, and this is the reason for the restriction of at least three samples along both dimensions in Theorem 1. We refer the reader to [15] and references therein for further information on this issue. Finally, note that, since the receiver needs to cover the entire band, the sampling rate should satisfy the Nyquist condition for the entire system bandwidth rather than the (much narrower) bandwidth of a single frequency-hop bin.

B. The Multidimensional-Folding (MDF) Algorithm

A variety of techniques have been developed for 2-D HR, e.g., in [4], [8], [12], and [14]. Among them, the MDF algorithm [14] can achieve the identifiability bound given by Theorem 1. For F small relative to the identifiability bound in Theorem 1, and moderate SNR and above, its performance is near-optimal, i.e., the covariance of the MDF estimates is close to the Cramér–Rao bound (CRB). Furthermore, MDF can resolve 2-D harmonic mixtures containing identical frequencies along one dimension [14]; hence, it can deal with frequency collisions.

Given (3), we may define $\mathbf{X} \in \mathbb{C}^{G\times L}$ with $\mathbf{X}(g,l) = x_{g,l}$, $\mathbf{A} \in \mathbb{C}^{G\times F}$ with $\mathbf{A}(g,f) = e^{j\omega_f(g-1)}$, $\mathbf{B} \in \mathbb{C}^{L\times F}$ with $\mathbf{B}(l,f) = e^{j\nu_f(l-1)}$, and a diagonal matrix $\mathbf{C} \in \mathbb{C}^{F\times F}$ with $\mathbf{C}(f,f) = c_f$. Then, the 2-D harmonic mixture in (3) can be written in matrix form (in the noisy case): $\mathbf{X} = \mathbf{A}\mathbf{C}\mathbf{B}^{\mathrm{T}} + \mathbf{W}$, where $\mathbf{W}$ is a white complex Gaussian-noise matrix. It is clear that (2) is of this form. The procedure for estimating (ω_f, ν_f, c_f), $f = 1, \ldots, F$, from $\mathbf{X}$ by the MDF algorithm is summarized below [14]

1) Let $G_1 = G_2 = (G+1)/2$ if G is odd, or $G_1 = G/2$ and $G_2 = (G+2)/2$ if G is even, and similarly choose L_1 and L_2 from L. Given $\mathbf{X}$, define a 2-D smoothed data matrix $\tilde{\mathbf{X}} \in \mathbb{C}^{G_1L_1\times G_2L_2}$ with typical element

$$\tilde{x}_{u,v} = x_{n_1+n_2-1,m_1+m_2-1} \tag{4}$$

for $1 \le n_i \le G_i$, $1 \le m_i \le L_i$, and $i = 1,\ 2$, where $u = (n_1-1)L_1 + m_1$, and $v = (n_2-1)L_2 + m_2$. It can be shown that the resulting matrix is $\tilde{\mathbf{X}} = (\mathbf{A}_1 \odot \mathbf{B}_1)\mathbf{C}(\mathbf{A}_2 \odot \mathbf{B}_2)^{\mathrm{T}} + \tilde{\mathbf{W}}$, where $\mathbf{A}_i$ is a submatrix of $\mathbf{A}$ consisting of its first G_i rows, and similar for $\mathbf{B}_i$, with $i = 1,\ 2$ [15]. $\tilde{\mathbf{W}}$ is the corresponding 2-D smoothed noise matrix. The Khatri–Rao product of two Vandermonde matrices has the property of almost-sure full rank. Hence, under the identifiability condition in Theorem 1, $\tilde{\mathbf{X}}$ is of rank F almost surely, even if $G < F$ or $L < F$ [15].

2) Let $\mathbf{Y} = \mathbf{J}\mathbf{X}^*\mathbf{J}$, where $\mathbf{J}$ is the permutation matrix with 1's on its main antidiagonal. Next, we construct a matrix $\tilde{\mathbf{Y}} \in \mathbb{C}^{G_1L_1\times G_2L_2}$ from $\mathbf{Y}$, following the same procedure used for the construction of $\tilde{\mathbf{X}}$ from $\mathbf{X}$. This is also known as "forward–backward averaging."

3) Compute the following singular value decomposition:

$$\begin{bmatrix} \tilde{\mathbf{X}} \\ \tilde{\mathbf{Y}} \end{bmatrix} = \begin{bmatrix} \mathbf{U}_1^{G_1L_1\times F} \\ \mathbf{U}_2^{G_1L_1\times F} \end{bmatrix} \mathbf{\Sigma}^{F\times F} (\mathbf{V}^{G_2L_2\times F})^{\mathrm{H}}. \tag{5}$$

4) Compute the eigenvectors $\mathbf{T}$ of $(\mathbf{U}_1)^\dagger \mathbf{U}_2$. In the noiseless case, $\mathbf{U}_1\mathbf{T} = (\mathbf{A}_1 \odot \mathbf{B}_1)\mathbf{\Pi}$, where $\mathbf{\Pi}$ is a nonsingular (column) permutation and scaling matrix that carries over from the solution of the eigenvalue problem (one can clearly reorder and scale the eigenvectors) [15]. This is not an issue, however, because ω_f and ν_f appear in the same column of $(\mathbf{A}_1 \odot \mathbf{B}_1)\mathbf{\Pi}$ (albeit not necessarily in column f, due to the arbitrary permutation) and are thus automatically paired; and arbitrary nonzero column scaling is immaterial, because the sought frequencies can be obtained by dividing suitably chosen elements of the said column. In the noisy case, the rich structure of the Khatri–Rao product of Vandermonde matrices can be exploited to average several point estimates of (ω_f, ν_f). After the 2-D frequencies are estimated, estimates of the associated complex amplitudes can be obtained by solving a simple linear least squares problem.

C. Joint Timing and Frequency Estimation: DP-2DHR

The key idea behind our proposed method of joint hop timing and frequency estimation is that between any two hypothesized systemwide hops, the data follow a 2-D harmonic model. Hence, for a hypothesized set of hops (that is, including all hops of all users in the system), 2-D HR methods can be used to estimate model parameters, and subsequently, model fit can be calculated. Best estimates are obtained when model fit is maximized. If one operates under an upper bound on the total (systemwide) number of hops, then system stage can be defined as the number of allowable hops, and state can be defined as the hop instant, hence, DP can be used to find the optimal hop sequence and associated model parameters per dwell. Note that this is different from assuming a bound on the number of hops on a per-user basis. The reason is that the computational complexity associated with the latter approach is exponential in the number of cochannel users. With d users and a budget of $K-1$ hops, define

$$\begin{aligned}
\mathbf{n} &= [n_1 \ \cdots \ n_{K-1}] \\
\boldsymbol{\alpha} &= \left[\alpha_1^{(0)} \ \cdots \ \alpha_1^{(K-1)} \ \cdots \ \alpha_d^{(0)} \ \ldots \ \alpha_d^{(K-1)}\right] \\
\boldsymbol{\beta} &= \left[\beta_1^{(0)} \ \cdots \ \beta_1^{(K-1)} \ \ldots \ \beta_d^{(0)} \ \ldots \ \beta_d^{(K-1)}\right] \\
\boldsymbol{\omega} &= \left[\omega_1^{(0)} \ \ldots \ \omega_1^{(K-1)} \ \ldots \ \omega_d^{(0)} \ \ldots \ \omega_d^{(K-1)}\right]
\end{aligned}$$

to be the vectors of hop timing, DOAs, complex frequency-dependent attenuations, and hop frequencies. Joint ML estimation of $\mathbf{n}$, $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, and $\boldsymbol{\omega}$ from $\mathbf{X}$ amounts to minimizing

$$J(\hat{\mathbf{n}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\omega}}) = \sum_{i=0}^{K-1} \|\mathbf{X}_i - \hat{\mathbf{X}}_i\|_{\mathrm{F}}^2 \tag{6}$$

over $\hat{\mathbf{n}}$, $\hat{\boldsymbol{\alpha}}$, $\hat{\boldsymbol{\beta}}$, $\hat{\boldsymbol{\omega}}$, where $\hat{\mathbf{X}}_i$ is the reconstructed 2-D harmonic mixture based on ML parameter estimates (DOAs, complex amplitudes, and carrier frequencies), obtained in each time segment defined by hypothesized $\hat{n}_i$ and $\hat{n}_{i+1}$, assuming a 2-D harmonic-mixture model for the received data during this segment. Since K will typically be higher than the true number of hops in the available samples, we include a "parking stage" in the DP program to account for the possibility of unused hops. In the presence of noise, however, DP will typically use any extra hops available to track minor noise-induced variations. Such variations can be relatively easily detected after DP, for frequencies before and after such hops will be approximately equal.

In practice, the ML estimates are approximated by applying the MDF algorithm to (2). The performance of the MDF algorithm is close to the CRB (and thus close to ML) only at relatively high SNR, and small rank relative to identifiability conditions. Thus, MDF only approximates the ML estimates, and the degree of approximation depends on the temporal sample length between two systemwide hops. The reasons that we use MDF are many: computational-complexity considerations (ML requires iterations and perhaps several initializations to avoid local minima), ability to resolve the signal parameters when there are identical frequencies along one dimension, and good performance. However, MDF is not ML; for this reason, the DP-MDF algorithm is not ML, but an approximation of ML.

From the MDF estimates, we form

$$\hat{\mathbf{x}}(n) = \sum_{r=1}^{d} \mathbf{a}(\hat{\theta}_r)\hat{\beta}_r \mathrm{e}^{j\hat{\omega}_r n} \tag{7}$$

for $n_i \leq n < n_{i+1}$; here, $\hat{\theta}_r = \mathrm{e}^{j2\pi\Delta\sin(\hat{\alpha}_r)}$, and the matrix $\hat{\mathbf{X}}_i$ is constructed from $\hat{\mathbf{x}}(n)$ in the same way that we constructed $\mathbf{X}_i$ in (2). Define $\Lambda_i[n_i, n_{i+1}-1]$, for $0 \leq i \leq K-1$, as the cost function for the time segment $n_i \leq n < n_{i+1}$

$$\Lambda_i[n_i, n_{i+1}-1] = \|\mathbf{X}_i - \hat{\mathbf{X}}_i\|_{\mathrm{F}}^2. \tag{8}$$

Furthermore, to solve the minimization problem in (6) by DP, we define

$$\Gamma_k(L) = \min_{\substack{n_1,\ldots,n_{k-1} \\ n_0=0, n_k=L+1}} \sum_{i=0}^{k-1} \Lambda_i[n_i, n_{i+1}-1] \tag{9}$$

where $0 < n_1 < \cdots < n_{k-1} < L$. Equation (9) can be viewed as the minimization problem of finding the best fit for a subset of the data of size $M \times (L+1)$ when a total number of $k-1$ hops is allowed. Hence, $\Gamma_K(N-1)$ is the minimum of $J(\hat{\mathbf{n}}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\omega}}, \hat{\boldsymbol{\phi}})$. From (9), a recursion for the minimum can be developed as

$$\Gamma_k(L) = \min_{n_{k-1}} \left(\Gamma_{k-1}(n_{k-1}-1) + \Lambda_{k-1}[n_{k-1}, L]\right). \tag{10}$$

This says that for a data matrix of size $M \times (L+1)$, the minimum error for k segments (i.e., $k-1$ hop instants) is the minimum error for the first $k-1$ segments that end at $n = n_{k-1}-1$, and the error contributed by the last segment from $n = n_{k-1}$ to $n = L$. The solution of the minimization of (6) is for $k = K$ and $L = N-1$, which yields the joint estimates of hop timing, DOAs, frequencies, and amplitudes of all users.

Assuming that the minimum length of a segment is three samples (since it is impossible to obtain valid frequency estimates from less than three samples using the MDF algorithm; see also Theorem 1), the procedure to compute the solution by the DP-2DHR algorithm is summarized in Table I. Note that frequencies and complex amplitudes of different segments pertaining to a particular user can be associated via their corresponding DOA parameters, since, for a single segment, frequency and DOA parameters pertaining to one user are paired up automatically by the MDF algorithm. Depending on different transmission schemes, the application of the DP-2DHR method may slightly vary as described in the following cases.

1) SFH with M-ary FSK modulation: Frequency changes due to baseband modulation are usually much smaller than those due to carrier hopping. Hence, symbol rate and hop rate can be obtained from the result of DP, and consequently, symbol recovery is possible.
2) SFH with M-ary PSK or M-ary QAM modulation: During one hop dwell, frequency is constant, but the complex amplitudes are different from symbol to symbol due to modulation (recall that for one hop dwell, the effect of channel on the complex amplitudes is constant). Hop timing can be detected from frequency change. Hence, symbol rate and hop rate are distinguishable from the result of DP.
3) SFH with GMSK modulation: A GMSK signal is not a pure exponential in one symbol period. However, narrowband GMSK can be well approximated by a pure exponential for our purpose. The robustness of the DP-2DHR method is tested for this case in Section V.
4) FFH: The DP-2DHR method is applicable for hop timing and hop-frequency-sequence estimation. However, additional information is needed for symbol detection, e.g., symbol period and symbol synchronization are required since the DP-2DHR can only provide chip synchronization in this case.

TABLE I
THE DP-2DHR ALGORITHM

1. **Initialization**
   Let $k = 1$, compute $\Gamma_k(L)$ for $L = 2, \ldots, N-3K+2$ using Eqns. (8) and (9).
2. **Recursion**
   For $2 \leq k \leq K-1$, compute $\Gamma_k(L)$ with $L = 3k-1, \ldots, N-3K+3k-1$, using Eqn. (10) with $3k-3 \leq n_{k-1} < L-2$; For $k = K$, compute $\Gamma_k(L)$ with $L = N-1$.
   For each L, denote the value of n_{k-1} that minimizes $\Gamma_k(L)$ as $n_{k-1}(L)$, and denote the corresponding $\hat{\boldsymbol{\alpha}}_{k-1}$, $\hat{\boldsymbol{\beta}}_{k-1}$, $\hat{\boldsymbol{\omega}}_{k-1}$ as $\hat{\boldsymbol{\alpha}}_{k-1}(L)$, $\hat{\boldsymbol{\beta}}_{k-1}(L)$, and $\hat{\boldsymbol{\omega}}_{k-1}(L)$, respectively.
3. **Backtracking**
   The maximum likelihood estimates of hop instants are obtained by using the backward recursion, i.e., $\hat{n}_i = n_i(\hat{n}_{i+1}-1)$, for $i = K-2, K-3, \ldots, 1$, initialized by $\hat{n}_{K-1} = n_{K-1}(N-1)$. Similarly, the corresponding DOA, amplitude, and frequency estimates of each segment can be obtained by their respective backward recursions.

In practical systems, the number of users is much less than the number of time samples, and the number of antenna elements usually ranges from 3 to 8. It can be shown that a good estimate of the complexity of the DP-2DHR algorithm is $\mathcal{O}(KN^5)$. There are several ways that this complexity can be reduced: 1) It is only during the initial acquisition period that the full complexity of the blind algorithm is needed. If frequencies hop at a regular rate, hop timing and hop period can be estimated by applying the DP-2DHR algorithm to a relatively short portion of a long data record, while frequency estimation for the remaining data can be accomplished by applying the MDF algorithm to predecided hop-free data blocks delimited by systemwide adjacent hop instants. This will reduce the complexity significantly. 2) The DP-2DHR algorithm may be simplified using standard approaches of reduced-complexity Viterbi decoding, such as path pruning based on metric thresholding, early path merging, etc. These will of course incur a performance loss, but if, e.g., the truncation parameter is appropriately chosen, the loss will be small. 3) If one has a reasonably good idea about the hop rates, the problem can be much simplified. If the hop code and hop timing of a user of interest are known, then one can dehop and obtain a model with much reduced noise and interference, since only interferers who collided with the particular user of interest within the observation interval will remain in the dehopped signal; and the receiver can cut down its bandwidth to the hopping-bin bandwidth in this case. 4) As a further alternative, simpler frequency-estimation techniques can be used in place of 2-D MDF. Clearly, there are many tradeoffs one may pursue.

In the development of the DP-2DHR, signal bandwidth is assumed to be known. In a practical blind-estimation scenario, the receiver may also lack knowledge of the signal bandwidth. Relative to the other unknowns (hop patterns, timing, and rates), it is simpler for the receiver to estimate the compound signal bandwidth, e.g., via energy detection. However, due to sampling-rate and noise-power considerations, an intercept receiver may only observe part of the spread bandwidth. In this case, the performance of DP-2DHR will be degraded due to bandwidth mismatch because users may hop in and out of the observed band, making it difficult to track across hops. Identifiability issues also become much more complicated in the presence of bandwidth mismatch, due to model-order variations.

IV. Extension of DP-2DHR

We now consider two interesting extensions of the results of Section III: First, the scenario where the signals are subject to multipath fading, and next, a case where multiple invariance (MI) arrays are available.

A. Multipath Channels With Small Delay Spread

The DP-2DHR method developed in Section III assumes single-path transmitter–receiver propagation for each FH user. When the signal bandwidth is greater than the channel-coherence bandwidth, channel effects due to multipath propagation cannot be ignored. Multipath reflections create fictitious sources in the spatial dimension, as well as unknown delay spread in the temporal dimension. Few techniques can be found in the literature on blind parameter estimation for FH signals in multipath channels, e.g., [7], which assumes a fixed but unknown header in each packet of the single FH user. In this section, we show that the DP-2DHR method can be extended to blindly estimate hop timing and frequency of multiple FH users in multipath channels with unknown but small delay spread.

Consider again the scenario shown in Fig. 1, where signals from d users are received at AP1. Suppose the signal of the rth user arrives at the ULA from L_r distinct paths due to multipath propagation, each with DOA α_{rl} (correspondingly, spatial frequency θ_{rl}), path attenuation β_{rl}, and time delay τ_{rl}, where $l = 1, \ldots, L_r$. At sampling time nT (T is normalized to 1), the baseband representation of the $M \times 1$ received-signal vector at the ULA output is

$$\begin{aligned}\mathbf{x}(n) &= \sum_{r=1}^{d}\sum_{l=1}^{L_r} \mathbf{a}\left(\theta_{rl}^{(p)}\right) \beta_{rl}^{(p)} s_r(n-\tau_{rl}) + \mathbf{w}(n) \\ &= \sum_{r=1}^{d}\sum_{l=1}^{L_r} \mathbf{a}\left(\theta_{rl}^{(p)}\right) \tilde{\beta}_{rl}^{(p)} \mathrm{e}^{j\omega_r^{(p)} n} + \mathbf{w}(n) \end{aligned} \qquad (11)$$

for $n = 0, \ldots, N-1$, where $\tilde{\beta}_{rl}^{(p)} = \beta_{rl}^{(p)} \mathrm{e}^{j\omega_r^{(p)}\tau_{rl}}$. Here, we assume that the delay spread for a given user is small so that time delay can be approximated by phase shift. Various channel measurements have shown that in an indoor environment the root mean square (rms) delay spread usually ranges from 20 to 50 ns [5], [20], compared to a maximum symbol rate of 2 Msps in 802.11 FHSS or Bluetooth. It has also been observed that the multipath arrivals group in clusters not only in time (e.g., one or two clusters as measured in an office building [20]) but also in angle (e.g., two to five clusters as measured in two university buildings [23]). Therefore, if we use the maximum-power multipath in each cluster to represent that cluster, then (11) incorporates a "specular" multipath model (few paths, parameterized by DOA and path loss with small delay spread), and is reasonable for an indoor environment.

Between any two systemwide consecutive hop instants, e.g., n_i and n_{i+1}, the received data can be expressed in matrix form $\mathbf{X}_i = \mathbf{A}_i \mathbf{B}_i \mathbf{S}_i^{\mathrm{T}} + \mathbf{W}_i$, which is a 2-D harmonic-mixture model that is essentially the same as (2), with the difference that the number of frequency components in such a time segment along each dimension is $\sum L_r$. Some frequency components in the time dimension are identical due to multipath reflection, but they can be dealt with by the MDF algorithm. Hence, DP-2DHR can be applied as before for joint hop timing and frequency estimation.

Note that frequencies and complex amplitudes of different segments pertaining to a particular path can be associated via their corresponding DOA parameters, since for a single segment, frequency, amplitude, and DOA parameters pertaining to one path are paired up automatically by the MDF algorithm. In addition, different paths pertaining to a particular user will result in different DOAs but identical hop-frequency sequence and hop timing (recall that time delay is treated as phase shift), hence, paths can be associated with users by hop sequences, which is a clustering problem and can be solved, e.g., by calculating the pairwise distance among all recovered hop sequences.

Notice that blind multiuser detection and identifiability analysis is more complicated in the presence of multipath with large delay spread.

B. MI Sensor Arrays

The principle of DP-2DHR can be extended to jointly estimate 2-D DOA (azimuth and elevation angles), hop timing, and frequency of multiple FH transmissions using an antenna array possessing MIs. An MI sensor array is composed of multiple identical subarrays displaced in the same or different directions. Several methods have been proposed for direction finding and/or other parameter estimation using MI sensor arrays, e.g., in [21], [24], [25], and [32]. An important assumption of these methods is that the incoming signals are narrowband, so that the propagation delay of the signals from one subarray to another can be approximated by phase shift. However, if the FH system under consideration is a wideband system, then the inherent frequency variability poses special difficulties for signal-parameter estimation, due to the fact that the phase shifts among subarrays are (wideband) frequency dependent. Nevertheless, these methods can still be applied to individual

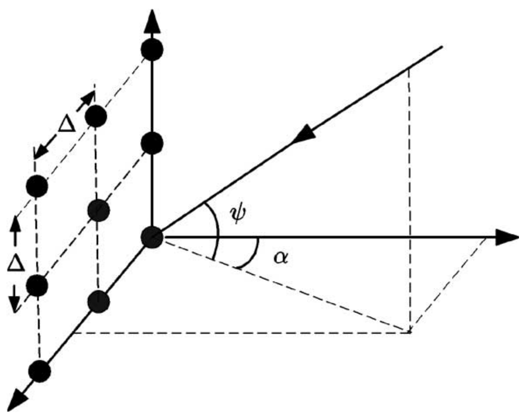
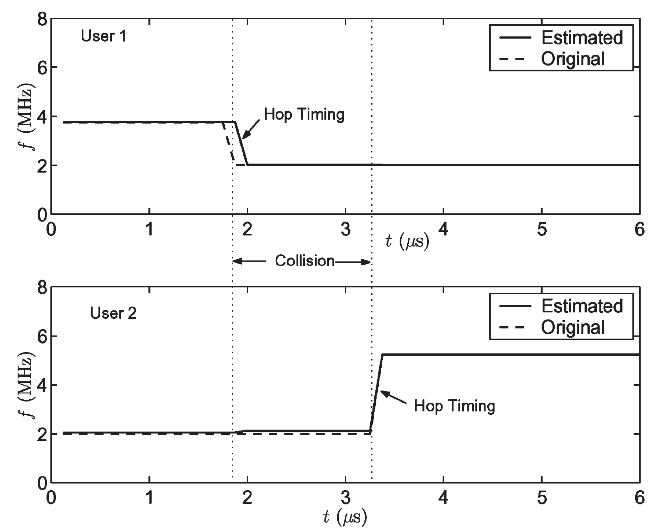


Fig. 3. A sensor array composed of overlapping subarrays.

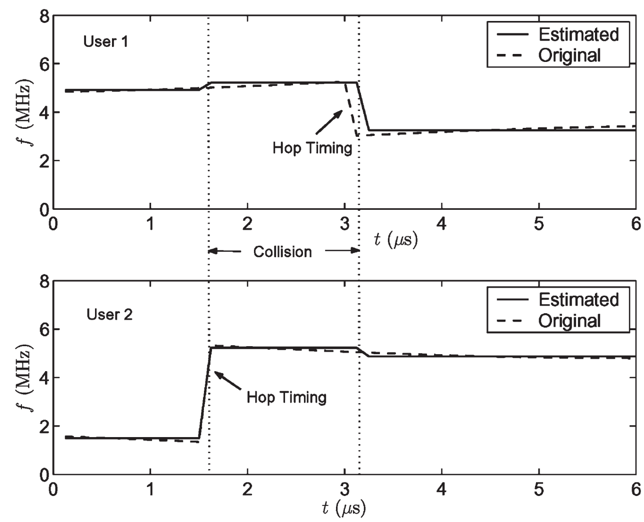


	True DOA	Estimated DOA		
		1st Seg.	2nd Seg.	3rd Seg.
user 1	12°	12.24°	13.23°	12.69°
user 2	17°	17.35°	16.10°	17.16°

Fig. 4. DP-2DHR: hop timing, frequency, and DOA estimation of two collided FH-FSK users (SNR $= 10$ dB).

hop-free segments if hop timing is known, since for a hop-free data segment, each of the signals impinging on the sensor array can be modeled as a narrowband signal. The direction finding and beamforming method proposed in [29] bypasses this problem by using a spatially colocated electromagnetic vector sensor. A single-user parameter-estimation method based on time-frequency distribution is proposed in [3].

Here, we extend the idea of DP-2DHR by coupling the DP principle with low-rank three-way decomposition of data collected from an MI sensor array. Suppose AP1 shown in Fig. 1 utilizes a 2-D antenna array, which is composed of H identical subarrays of m sensors, each displaced in different directions. An example of such an array is shown in Fig. 3. The total number of sensors is M, and, in general, $m + H - 1 \leq M \leq mH$. The left bound is met for subarrays that share $m - 1$ elements, and the right bound is met for subarrays that have



	True DOA	Estimated DOA		
		1st Seg.	2nd Seg.	3rd Seg.
user 1	12°	12.34°	12.22°	12.58°
user 2	17°	17.82°	16.54°	16.75°

Fig. 5. DP-2DHR: hop timing, frequency, and DOA estimation of two collided FH-GMSK users (SNR $= 10$ dB).

no overlap. Though users' carrier frequencies are hopped over a wide frequency band, between any two systemwide consecutive hop instants, e.g., n_i and n_{i+1}, the discrete-time baseband equivalent model for the array output can still be written as an $M \times (n_{i+1} - n_i)$ matrix

$$\mathbf{X}_i = \mathcal{A}_i \mathbf{S}_i^{\mathrm{T}} + \mathbf{W}_i \tag{12}$$

where $\mathcal{A}_i = [\mathbf{a}(\alpha_1, \psi_1) \cdots \mathbf{a}(\alpha_d, \psi_d)]$, and α_r, ψ_r are azimuth and elevation angles. The frequency-dependent complex amplitude (due to path attenuation) is absorbed into $\mathcal{A}_i$. Let $\mathbf{J}_h$ denote the $m \times M$ selection matrix that extracts the m rows corresponding to the hth subarray; then, it holds that [25]

$$\mathbf{Y}_i = \begin{bmatrix} \mathbf{J}_1 \\ \vdots \\ \mathbf{J}_H \end{bmatrix} \mathbf{X}_i = \begin{bmatrix} \mathbf{A}_i \mathbf{\Phi}_i^{(1)} \\ \vdots \\ \mathbf{A}_i \mathbf{\Phi}_i^{(\mathrm{H})} \end{bmatrix} \mathbf{S}_i^{\mathrm{T}} + \mathbf{V}_i \tag{13}$$

where the $m \times d$ matrix $\mathbf{A}_i$ is the response of subarray 1 (reference), and $\mathbf{\Phi}_i^{(h)}$ is a $d \times d$ diagonal matrix of phase shifts, which is a function of signal parameters (DOA and frequency) and the displacement of the hth subarray relative to the reference, with $\mathbf{\Phi}_i^{(1)} = \mathbf{I}$. $\mathbf{V}_i$ is the corresponding noise matrix. Define an $H \times d$ matrix $\mathbf{\Phi}_i$ such that its hth row consists of the diagonal elements of $\mathbf{\Phi}_i^{(h)}$; then, (13) can be rewritten as

$$\mathbf{Y}_i = (\mathbf{\Phi}_i \odot \mathbf{A}_i) \mathbf{S}_i^{\mathrm{T}} + \mathbf{V}_i. \tag{14}$$

For given n_i and n_{i+1}, the objective is to blindly estimate 2-D directions α_r and θ_r, as well as frequency $\omega_r^{(p)}$ from $\mathbf{Y}_i$, for $r = 1, \ldots, d$. This problem can be solved by several techniques, e.g., in [21], [24], and [25]. A key observation

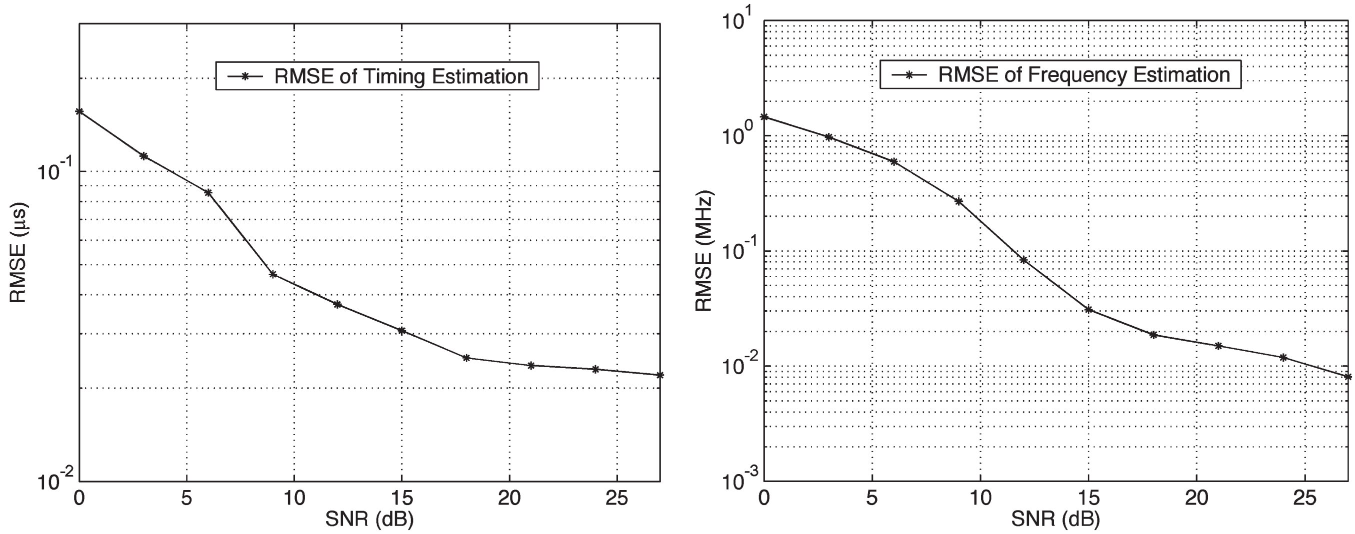


Fig. 6. DP-2DHR: rmse of hop timing and frequency estimation versus SNR in the presence of collisions.

is that with proper dimensioning and under certain relatively mild conditions, (14) is in fact a low-rank trilinear (three-way) model that exhibits strong identifiability properties and can be estimated via well-established iterative least squares algorithms [21]. Low-rank three-way array decomposition is unique under a relatively mild ranklike condition [10]. The identifiability of model (14) is established in [21].

In particular, trilinear alternating least squares (TALS) can be used to estimate $\mathbf{A}_i$, $\mathbf{\Phi}_i$, and $\mathbf{S}_i$ from the noisy observations $\mathbf{Y}_i$. The basic idea of ALS is to update matrices one by one in an alternating fashion during each iteration, conditioned on previously obtained estimates for the remaining matrices [21]. Upon convergence of TALS, $\mathbf{A}_i$, $\mathbf{\Phi}_i$, and $\mathbf{S}_i$ are estimated up to scaling and common permutation of columns. The azimuth and elevation angles can then be estimated via simple division from $\mathbf{\Phi}_i$, and the temporal frequencies can be estimated from $\mathbf{S}_i$ via single one-dimensional (1-D) HR techniques (e.g., periodogram) or simple division. Since the permutation of columns is common to all three matrices, $(\alpha_r, \psi_r, \omega_r^{(p)})$ will be paired up automatically by TALS.

Notice that both $\mathbf{A}_i$ and $\mathbf{S}_i$ in (14) are Vandermonde. This constraint can be incorporated into the iteration process of TALS to expedite convergence and improve estimation performance. Joint timing and frequency estimation is achieved by coupling DP with TALS (DP-TALS).

Remark 1: There are other possible extensions of the proposed algorithms. For example, the DP principle can also be used to cope with uncalibrated array manifolds by replacing MDF with estimation of signal parameters via rotational invariance techniques (ESPRIT) [19], but then, association of dwells is an issue.

V. Simulation Results

A. The DP-2DHR Algorithm for Collision Resolution

In this section, we present the simulation results to demonstrate the proposed DP-2DHR for joint hop timing and frequency estimation in the presence of frequency collisions. Unless otherwise stated, two FH users with DOAs [12°, 17°] are simulated, each hopping with different hop timing. The receiver array has $M = 6$ antennas, with baseline separation of $\lambda/2$ at $f_{\mathrm{c}} = 1$ GHz. With $M = 6$, the array has a 3-dB beamwidth of about 28° so that the two sources, separated by 5°, are not directly resolvable. A hopping frequency band of bandwidth 8 MHz is occupied by 32 frequency channels with 0.25-MHz channel spacing. The received signal is well modeled as narrowband. For simplicity of illustration, hop rate is set the same as symbol rate (125 Kb/s). At the receiver, the complex antenna outputs are sampled at a rate of 8 MHz after down conversion, and $N = 48$ complex samples are collected at each antenna, resulting in a 6-μs-long analysis window, hence, each user hops at most once within this window. Throughout the simulation, SNR is defined as [cf., (1)]

$$\mathrm{SNR} := 10\log_{10}\left(\frac{\|\mathbf{X}-\mathbf{W}\|_{\mathrm{F}}^2}{MN\sigma^2}\right) \tag{15}$$

where the noise variance $\sigma^2 = N_0 B$, and B is the processing signal bandwidth.

Test A1 (FH-FSK, $\mathrm{SNR} = 10$ dB): An example for the FH-FSK case is shown in Fig. 4. In this example, two binary FSK (BFSK) users begin in different bins; then, user 1 hops to the same bin as user 2; later, user 2 hops out of his original bin and into a new bin. This gives three segments: the first and the third without collision, and the second with collisions. Fig. 4 gives the DP-2DHR results of DOA estimation for those three segments, and the corresponding results of hop timing and frequency estimation for the two users. SNR is 10 dB. We assume that any mobility-induced changes in DOA are negligible within the analysis window. Thus, varying hop frequencies are associated with different users via their corresponding window-invariant DOA parameters. The results show that DOA, hop timing, and frequency estimates are close to the respective true values even in the presence of collisions. They also demonstrate that good estimates can be obtained, based on measurements of duration less than one symbol period; this implies that the algorithm is capable of collision resolution at moderate to heavy loads.

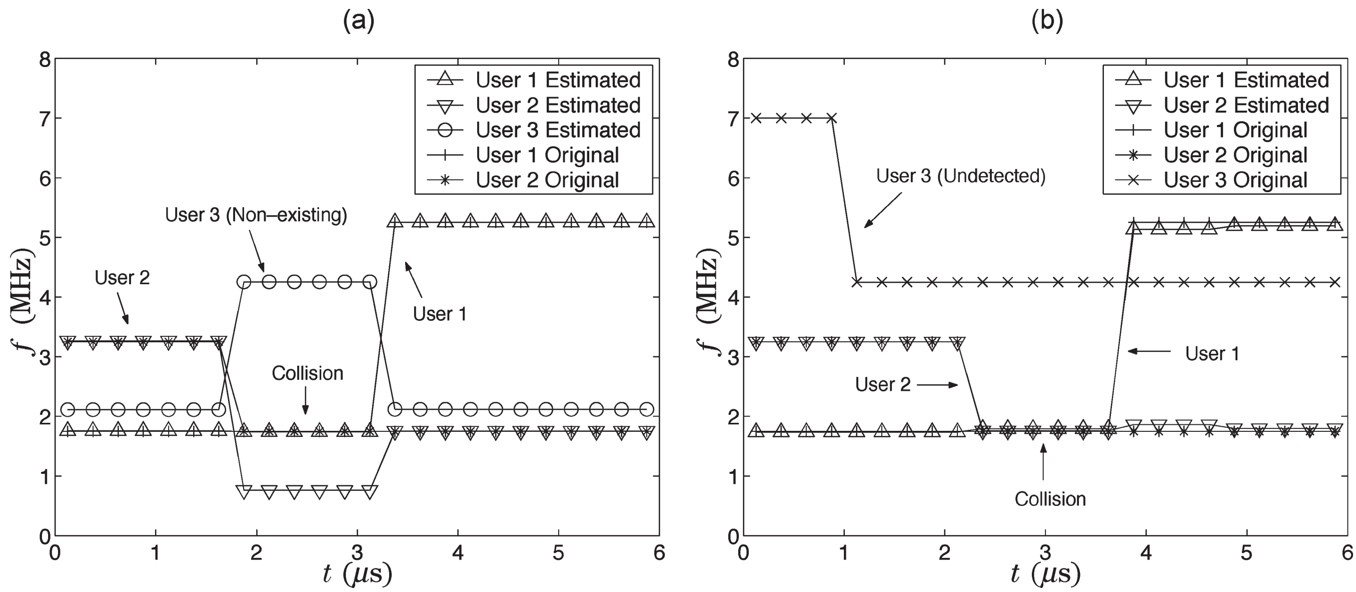
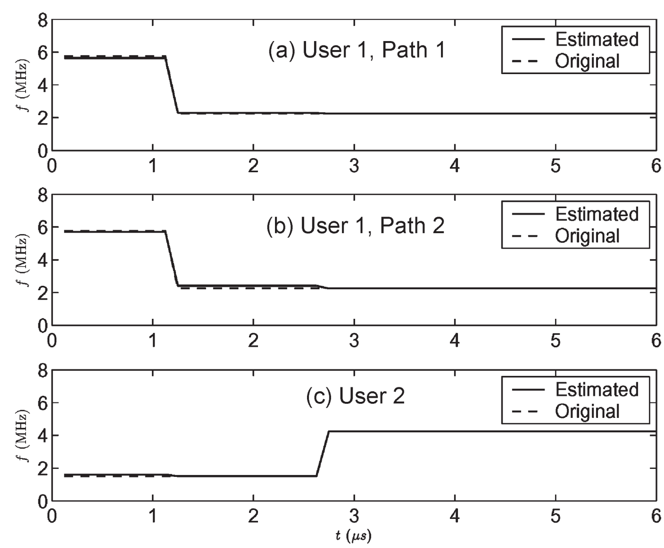


Fig. 7. DP-2DHR: collision resolution in the presence of model-order mismatch at SNR = 10 dB. (a) The number of users is overestimated (estimated value 3 versus true value 2); (b) The number of users is underestimated (estimated value 2 versus true value 3). In both cases, the number of systemwide hops is assumed to be correctly estimated.

Test A2 (FH-GMSK, SNR = 10 dB): Another case of interest is FH-GMSK, which is used in IEEE 802.11. GMSK is a special case of continuous phase modulation (CPM), and does not adhere to the harmonic model that we have assumed here, which is valid for FSK and PSK. We test the robustness of the DP-2DHR algorithm by applying it to track two collided FH-GMSK signals. We set $BT = 0.5$, where B is the Gaussian filter bandwidth, and T is the symbol period. An example is shown in Fig. 5, where SNR is 10 dB. The center segment indicates a frequency collision. Notice that the original GMSK signal's frequency is changing continuously (with a shape similar to the Gaussian pulse), but it can be approximated by an exponential since it is a narrowband signal. As can be seen from Fig. 5, the DP-2DHR algorithm is able to resolve collisions with GMSK modulations.

Test A3: Fig. 6 depicts the empirical root mean square error (rmse) of DP-2DHR hop timing and frequency estimates in the presence of collisions. For each realization, each of the two FH-FSK users hops once within the observation window. Hop timing is randomly generated, and frequencies are also randomly selected from the 32 candidate bins with the constraint that there is always one collision in the three hop-free segments. The rmse-versus-SNR performance shown in Fig. 6 indicates that the DP-2DHR algorithm performs quite well even in the low-SNR regime, given the fact that the signals are tracked in a situation where hop pattern, rate, and timing are all unknown.

Test A4: In this example, we illustrate the DP-2DHR for collision resolution in case of model-order overestimation and underestimation. Differences in signal power may induce errors in model-order estimation. In Fig. 7(a), DP-2DHR is applied assuming that three users are detected while two FH-FSK users with DOA = $[12^\circ\ 17^\circ]$ are actually present (the two users have a 10-dB difference in power). In Fig. 7(b), DP-2DHR is applied assuming that two users are detected while three FH-FSK users from DOA = $[12^\circ\ 40^\circ\ 17^\circ]$ are actually present, with the power of the third user being 10-dB less than that of user 1 (users 1 and 2 have equal powers). In both cases, the number of systemwide hops is assumed to be correctly estimated. It can be seen from Fig. 7 that the result is very good in the underestimated case, and even in the overestimated case, the hop instants are correctly detected and all but one frequencies are correctly estimated for users 1 and 2. Similar results are obtained even



	True DOA	Estimated DOA		
		1st Seg.	2nd Seg.	3rd Seg.
Path 1-1	6°	4.67°	7.48°	6.33°
Path 1-2	14°	12.25°	14.85°	13.55°
Path 2-1	25°	25.50°	24.91°	24.17°

Fig. 8. DP-2DHR: hop timing, frequency, and DOA estimation of two FH-FSK users in the presence of multipath.

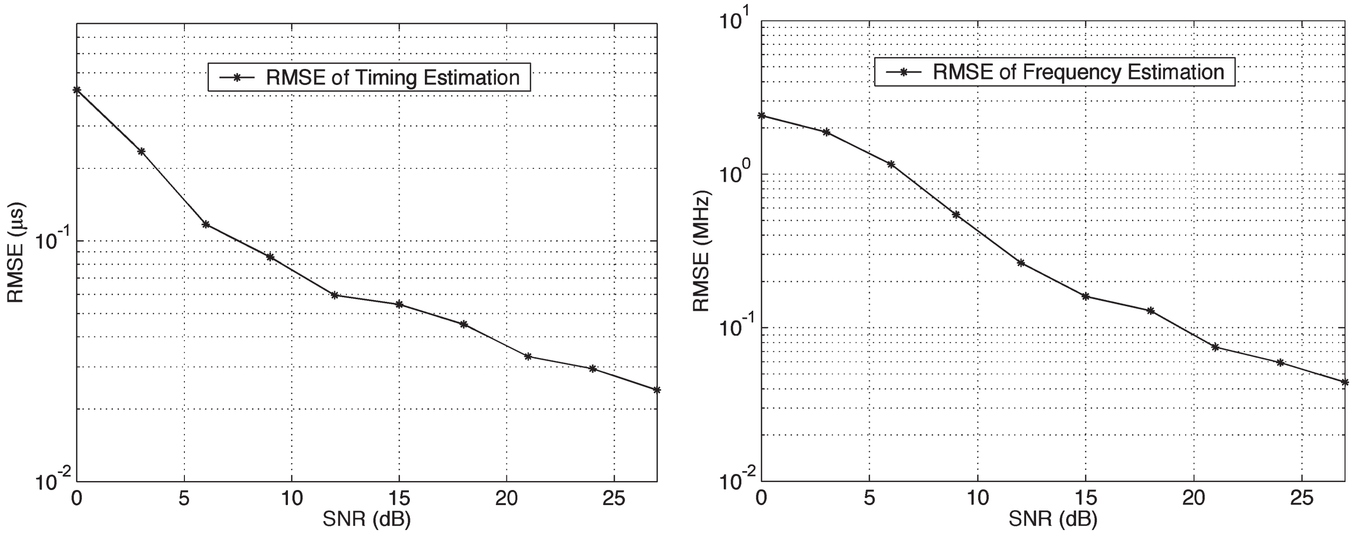


Fig. 9. DP-2DHR: rmse of hop timing and frequency estimation versus SNR in the presence of multipath.

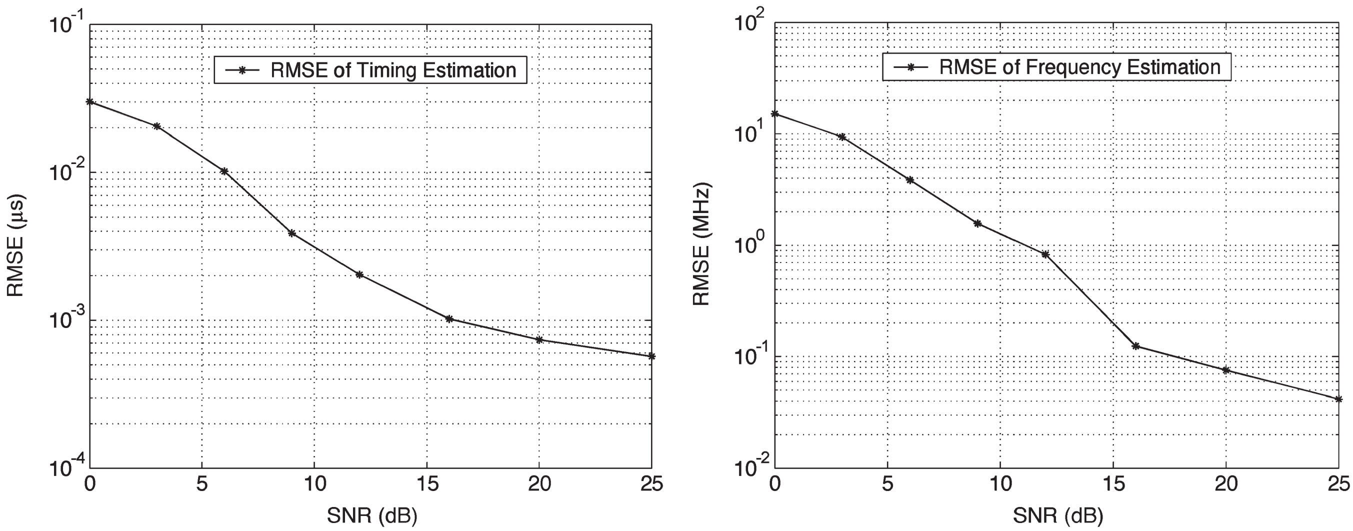


Fig. 10. DP-TALS: rmse of hop timing and frequency estimation versus SNR using a 2-D antenna array.

for moderately overestimated hop budgets (e.g., twice the actual number of hops).

B. DP-2DHR in the Presence of Multipath Channels

We also tested the DP-2DHR algorithm for hop timing and frequency estimation in multipath channels. Suppose the signal of one user propagates through two paths with DOA $= [6°, 14°]$ with the second path delayed 0.2 μs, and the other user has a single path at DOA $= 25°$. Other parameters are similar to those in Section V-A. For every realization of the Monte Carlo simulation, hop timing and frequencies are randomly generated, while no specific collision is introduced since "self-collisions" are inherent in this multipath scenario. Here, the delay spread is much less than the symbol period (8 μs), as assumed in the preceding derivation.

Test B1 (FH-FSK, SNR $= 10$ dB): Fig. 8 is an example of the hop timing, frequency, and DOA estimation for the three paths, where "Path 1–2" denotes the second path of user 1. Again, varying hop frequencies are associated with different paths via their corresponding window-invariant DOA parameters. The results show that hop timing and frequency estimates are close to their respective true values. Fig. 8 also indicates that paths 1 and 2 pertain to the same user since they have essentially the same hop timing and frequency sequence. Similar results have been obtained for GMSK-modulated signals, which are omitted here.

Test B2: Fig. 9 plots the rmse of hop timing and frequency estimation of DP-2DHR in the presence of multipath propagation. The results indicate that the DP-2DHR algorithm performs well in multipath channels, given the fact that the signals are tracked in a situation where hop code, rate, timing, and multipath delay are all unknown.

C. DP-TALS With MI Arrays

Test C1: A rectangular array of size 2×6 is used in this test, comprising four overlapping uniform linear subarrays of five sensors each. The spacing Δ is half wavelength at $f_c =$ 2 GHz. Two BFSK modulated users with 2-D DOA (elevation

α, azimuth ψ) angles of (10°, 6°) and (17°, 12°), hop at different timing. The hopping bandwidth is 80 MHz with 1-MHz channel spacing. Both symbol period and hop dwell are 0.8 μs. The received signal is sampled at 80 MHz after down conversion, and $N = 48$ samples are collected at each sensor. For each realization of the Monte Carlo simulation, each of the two FH-FSK users hops once within the observation window. Hop timing and frequencies are randomly generated. The rmse of DP-TALS versus SNR curves in Fig. 10 demonstrate that the algorithm performs quite well for a wide range of SNRs. However, we note that the complexity of DP-TALS is higher than that of DP-2DHR due to the iterative nature of TALS.

VI. Conclusion

We proposed a joint hop-timing-and-frequency-estimation method based on the principle of DP coupled with 2-D HR or low-rank trilinear decomposition. The multiuser detection method is capable of unraveling multiple FH signals in the presence of collisions and unknown hop patterns without retransmission. It remains operational even with multiple (unknown) hop rates, frequency offsets, and asynchronism. Simulation results show that it is also robust to model mismatch with respect to modulation techniques.

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Xiangqian Liu (S'98–M'02) received the B.S. degree in electrical engineering from Beijing Institute of Technology, Beijing, China, in 1997, the M.S. degree in electrical engineering from University of Virginia, Charlottesville, in 1999, and the Ph.D. degree in electrical engineering from University of Minnesota, Minneapolis, in 2002.

Since 2002, he has been an Assistant Professor of Electrical and Computer Engineering at University of Louisville, Louisville, KY. His current research interests are in the areas of wireless communications and signal processing, including multiuser detection, frequency estimation, spread-spectrum communication systems, and sensor-array processing, as well as multiway analysis.

Nicholas D. Sidiropoulos (S'90–M'92–SM'99) received the Diploma in electrical engineering from the Aristotelian University of Thessaloniki, Greece, in 1988, and the M.S. and Ph.D. degrees in electrical engineering from the University of Maryland at College Park (UMCP), in 1990 and 1992, respectively.

From 1988 to 1992, he was a Fulbright Fellow and a Research Assistant at the Institute for Systems Research (ISR) of the University of Maryland. From September 1992 to June 1994, he served his military service as a Lecturer in the Hellenic Air Force Academy. From October 1993 to June 1994, he also was a member of the Technical Staff, Systems Integration Division, G-Systems Ltd., Athens—Greece. He has been a Postdoctoral Fellow (1994–1995) and Research Scientist (1996–1997) at ISR-UMCP, an Assistant Professor in the Department of Electrical Engineering at the University of Virginia (1997–1999), and an Associate Professor in the Department of Electrical and Computer Engineering at the University of Minnesota—Minneapolis (2000–2002). He is currently a Professor in the Telecommunications Division of the Department of Electronic and Computer Engineering at the Technical University of Crete, Chania-Crete, Greece, and Adjunct Professor at the University of Minnesota. He is an active consultant for industry in the areas of frequency-hopping systems and signal processing for digital subscriber line (xDSL) modems. His current research interests are primarily in signal processing for communications, and multiway analysis.

Dr. Sidiropoulos is a member of the Signal Processing for Communications Technical Committee (SPCOM-TC), and the Sensor Array and Multichannel processing Technical Committee (SAM-TC) of the IEEE SP Society, and Associate Editor for IEEE TRANSACTIONS ON SIGNAL PROCESSING (2000–present). During 2000–2002, he also served as Associate Editor for IEEE SIGNAL PROCESSING LETTERS. He received the National Science Foundation (NSF)/CAREER award (Signal Processing Systems Program) in June 1998, and an IEEE Signal Processing Society best paper award in 2001.

Ananthram Swami (SM'96) received the B.Tech. degree from Indian Institute of Technology (IIT), Bombay; the M.S. degree from Rice University, Houston; and the Ph.D. degree from the University of Southern California (USC), all in electrical engineering.

He has held positions with Unocal Corporation, USC, CS-3, and Malgudi Systems. He is currently with the US Army Research Laboratory, Adelphi, MD, where he is a Fellow. He was a Statistical Consultant to the California Lottery, developed a Matlab-based toolbox for non-Gaussian signal processing, and has held Visiting Faculty positions at Institut National Polytechnique (INP), Toulouse, France. He has taught short courses for industry, and occasionally teaches courses on communication theory and signal processing. His work is in the broad area of signal processing and communications.

Dr. Swami is a member and Vice-chair of the IEEE Signal Processing Society's TC on Signal Processing for Communications, an Associate Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, and of the IEEE TRANSACTIONS ON SIGNAL PROCESSING, and a member of the Editorial Board of the IEEE *Signal Processing Magazine*. He has served as an Associate Editor (AE) for IEEE SIGNAL PROCESSING LETTERS, and IEEE TRANSACTIONS ON CIRCUITS & SYSTEMS—II. He was Coorganizer and Cochair of the 1993 IEEE-SPS HOS Workshop, the 1996 IEEE-SPS SSAP Workshop, and the 1999 ASA-IMA Workshop on Heavy-Tailed Phenomena. He is a Coguest Editor of a 2004 special issue of the IEEE *Signal Processing Magazine* on "Signal Processing for Networking."