

Multicast beamforming and admission control for UMTS-LTE and 802.16e

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Parts of the talk

- Part I:
 - QoS + max-min fair multicast beamforming
- Part II:
 - Joint QoS multicast beamforming and admission control

Motivation

- Multicasting increasingly important (network TV, streaming media, software updates, network management)
- Increasingly over wireless for last hop
- PHY-layer multicasting – exploits wireless “broadcast advantage” + CSI-T [SidDavLuo:04-06]
- Complements packet-level multicasting → higher efficiency

Motivation: E-MBMS / UMTS-LTE

- Evolved Multimedia Broadcast/Multicast Service (E-MBMS) in the context of 3GPP / UMTS-LTE
- Motorola Inc., “Long Term Evolution (LTE): A Technical Overview,” Technical White Paper:

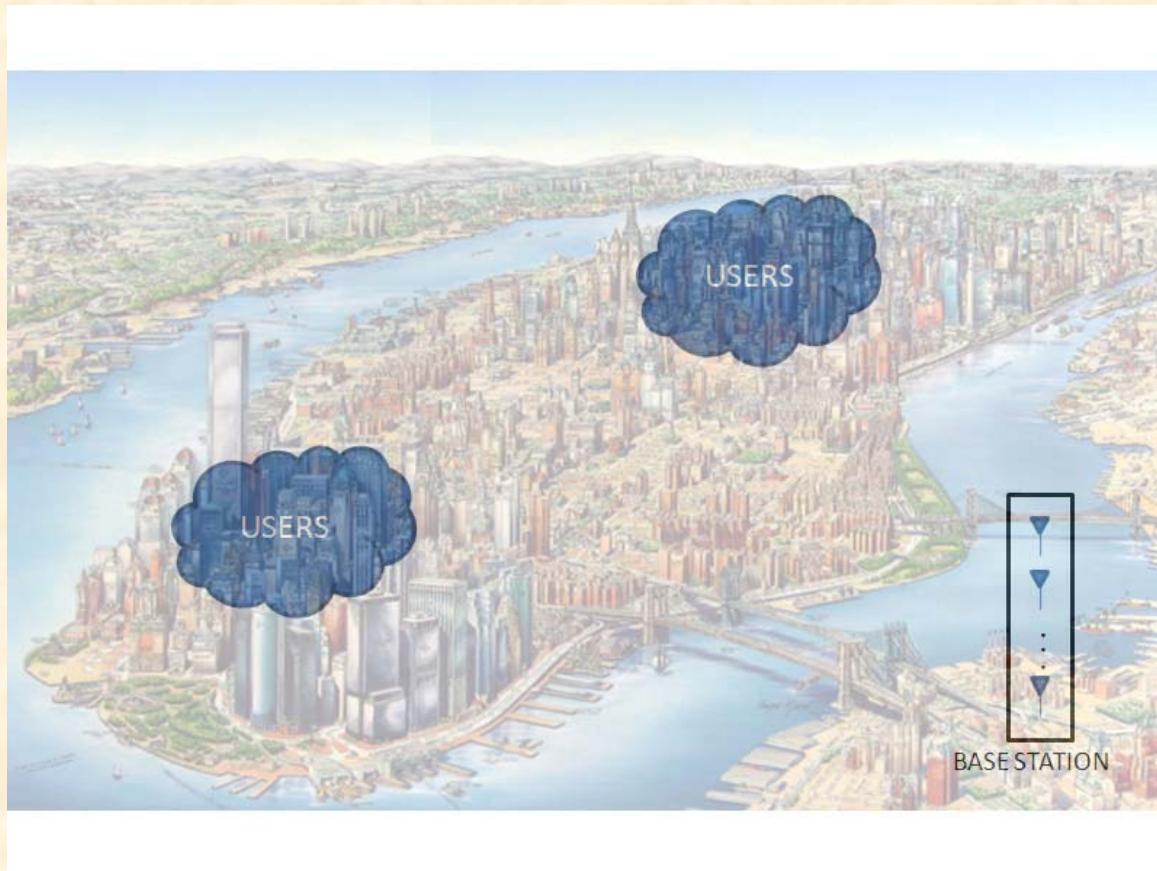
<http://business.motorola.com/experiencelte/pdf/LTE%20Technical%20Overview.pdf>

↓
2. Multicast Traffic Channel (MTCH)
(DL point-to-multipoint channel for transmission of MBMS data)

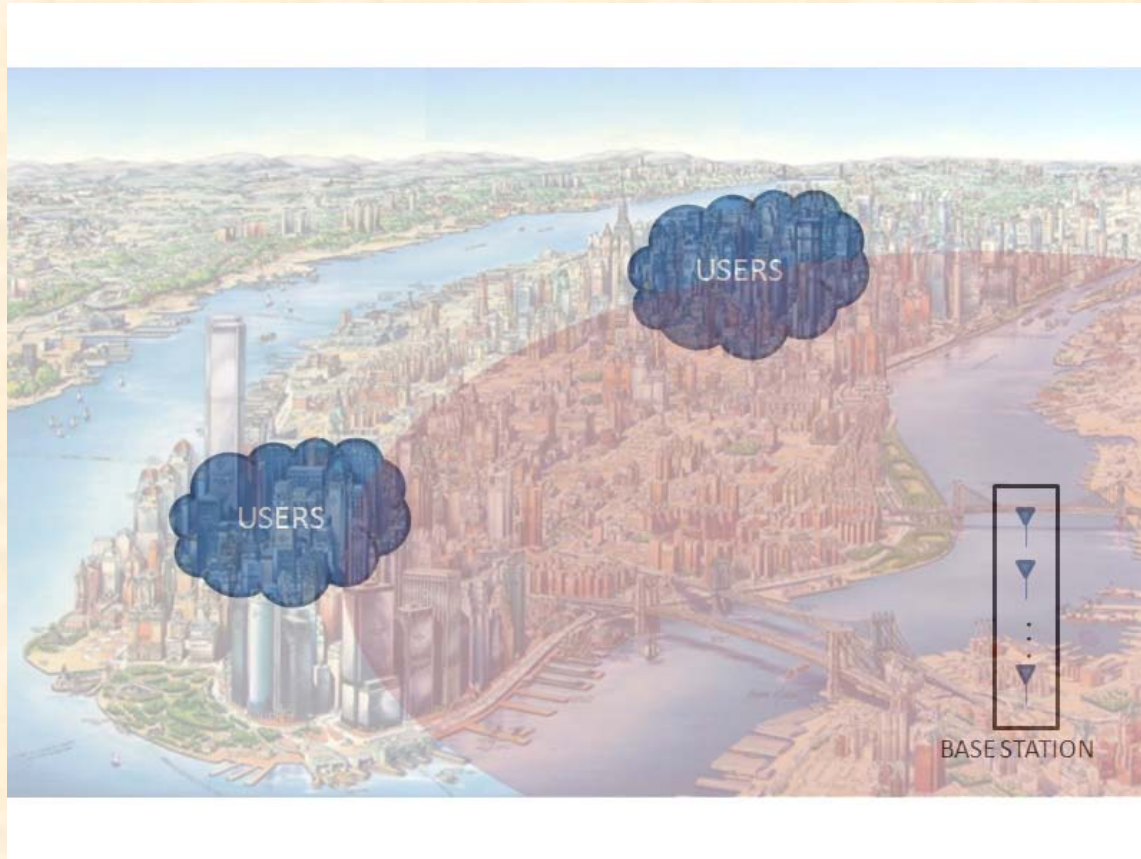
EVOLVED MULTICAST BROADCAST MULTIMEDIA SERVICES (E-MBMS)

There will be support for MBMS right from the first version of LTE specifications. However, specifications for E-MBMS are in early stages. Two important scenarios have been identified for E-MBMS: One is single-cell broadcast, and the second is MBMS Single Frequency Network (MBSFN).

Prelude



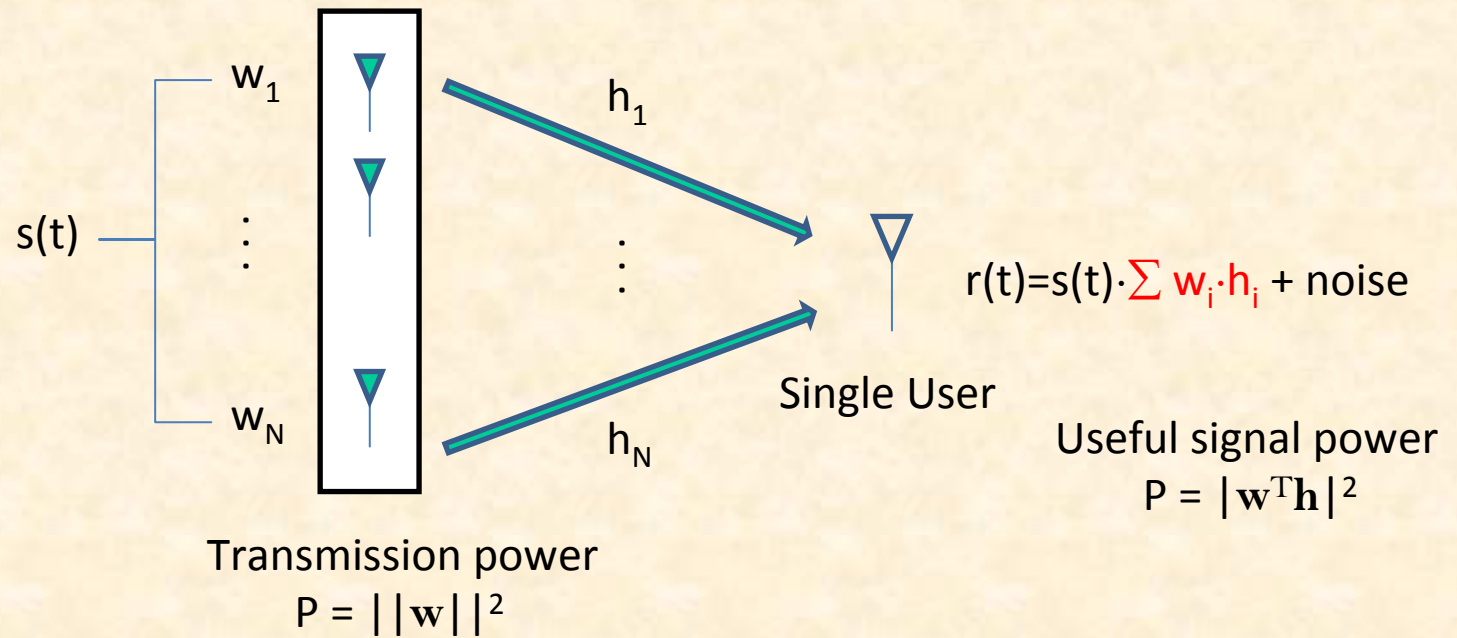
Broadcast



Multicast beamforming



Beamforming



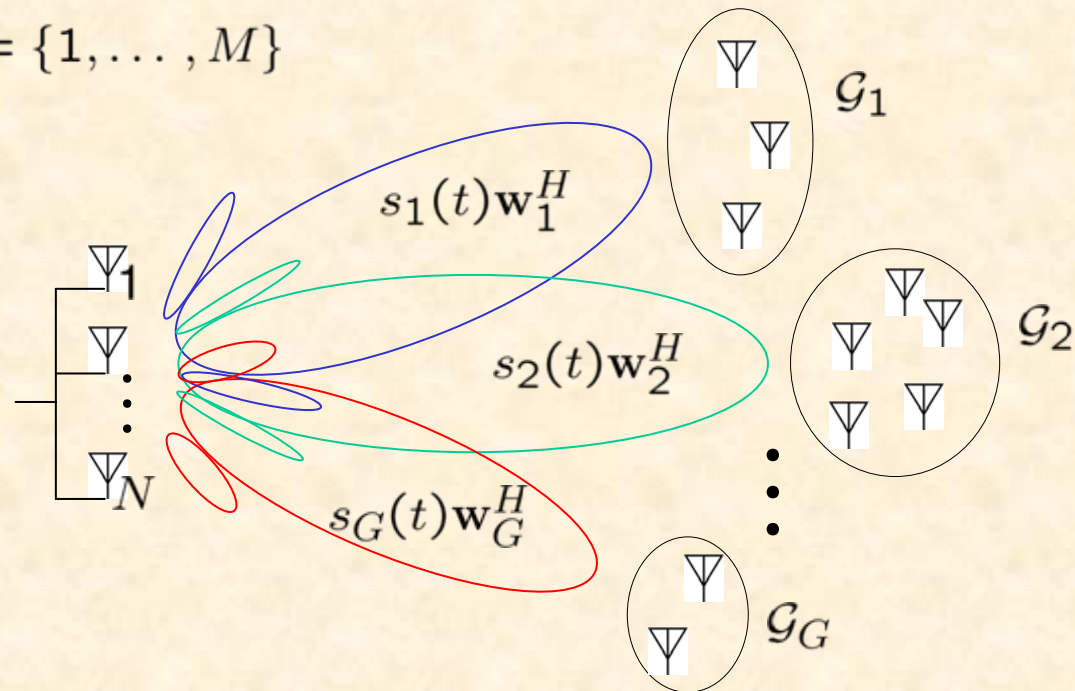
Part I: Transmit Beamforming for Multicasting

- Joint work w/ Tim Davidson, Tom Luo, Lefteris Karipidis
- Problem statement:
 - Transmit beamforming for multicasting to multiple co-channel groups
- QoS formulation
- NP-hardness
- Multicast power control
- Max-min-fair version
- The Vandermonde case
- Robust formulations

Problem Setup

- Downlink Transmission: BS has N antenna elements
- M single-antenna intended mobile receivers
- $1 \leq G \leq M$ co-channel multicast groups \mathcal{G}_k , $k \in \{1, \dots, G\}$,
 $\mathcal{G}_k \cap \mathcal{G}_l = \emptyset$, $l \neq k$, $\cup_k \mathcal{G}_k = \{1, \dots, M\}$
- $s_k(t)$: modulated signal,
 sent to group \mathcal{G}_k ,

$$\sum_{k=1}^G s_k(t) \mathbf{w}_k^H: \text{Tx signal}$$



QoS formulation

- Optimal joint design of transmit beamformers (full CSI at Tx)
- QoS formulation: Minimize total Tx power, subject to meeting prescribed lower bounds on the received SINRs

$$\begin{aligned} \mathcal{I} : \\ \min_{\{\mathbf{w}_k \in \mathbb{C}^N\}_{k=1}^G} \sum_{k=1}^G \|\mathbf{w}_k\|_2^2 \\ \text{s.t. : } \frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{l \neq k} |\mathbf{w}_l^H \mathbf{h}_i|^2 + \sigma_i^2} \geq \gamma_i, \quad \forall i \in \mathcal{G}_k, \quad \forall k \in \{1, \dots, G\} \end{aligned}$$

- Special cases:
 - multiuser downlink ($G = M$) is SOCP (Bengtsson & Ottersten);
 - broadcasting ($G = 1$) (Sidiropoulos, Davidson, Luo)
 - middle ground

Single multicast group ($G=1$)

$$\begin{aligned} \min_{\mathbf{w} \in \mathbb{C}^N} \quad & \|\mathbf{w}\|_2^2 \\ \text{s.t.} \quad & \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \geq \gamma_i, \quad \forall i \end{aligned}$$

- Seems **benign** ...
- ... but non-convex, and in fact NP-hard!
- Contains *partition* (Sidiropoulos, Davidson, Luo '06)

Hence ...

$$\mathcal{I} :$$

$$\min_{\{\mathbf{w}_k \in \mathbb{C}^N\}_{k=1}^G} \sum_{k=1}^G \|\mathbf{w}_k\|_2^2$$

$$\text{s.t. : } \frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{l \neq k} |\mathbf{w}_l^H \mathbf{h}_i|^2 + \sigma_i^2} \geq \gamma_i, \forall i \in \mathcal{G}_k, \forall k \in \{1, \dots, G\}$$

→ NP-hard in general ☹

Recasting to isolate non-convexity

- Equivalent reformulation for $\mathbf{Q}_i := \mathbf{h}_i \mathbf{h}_i^H$ and $\mathbf{W}_k := \mathbf{w}_k \mathbf{w}_k^H$

$$\begin{aligned}
 \mathcal{R} : \\
 & \min_{\{\mathbf{W}_k \in \mathbb{C}^{N \times N}\}_{k=1}^G, \{s_i \in \mathbb{R}\}_{i=1}^M} \sum_{k=1}^G \text{trace}(\mathbf{W}_k) \\
 \text{s.t. : } & \text{trace}(\mathbf{Q}_i \mathbf{W}_k) - \gamma_i \sum_{l \neq k} \text{trace}(\mathbf{Q}_i \mathbf{W}_l) - s_i = \gamma_i \sigma_i^2, \\
 & \forall i \in \mathcal{G}_k, \forall k \in \{1, \dots, G\}, \\
 & s_i \geq 0, \forall i \in \{1, \dots, M\}, \\
 & \mathbf{W}_k \succeq \mathbf{0}, \forall k \in \{1, \dots, G\} \\
 & \text{rank}(\mathbf{W}_k) = 1, \forall k \in \{1, \dots, G\}
 \end{aligned}$$

- \mathcal{R} is *SDP*: lin. cost func. & M lin. eq., M nonneg., G psd constraints
- Lagrange bi-dual interpretation

Algorithm [KarSidLuo:TSP08]

- *Relaxation*: Solve the SDP Problem \mathcal{R} , denote solution $\{\mathbf{W}_k\}_{k=1}^G$
- *Randomization / Scaling Loop*: For each k , generate a vector in the span of $\sqrt{\mathbf{W}_k}$, using the Gaussian randomization technique, and solve multicast power control problem (LP) for given configuration;
If feasible, then feasible solution for original problem
- Repeat, select best configuration (minimum Tx power)
- If $\text{rank}(\mathbf{W}_k) = 1, \forall k, \Rightarrow$ Problem \mathcal{R} equiv to \mathcal{I}
- Quality of approximate solution: $\frac{\min(\sum_{k=1}^G \beta_k p_k)}{\sum_{k=1}^G \text{trace}(\mathbf{X}_k)}$

Multi-group Multicast Power Control

- If Problem \mathcal{I} feasible \Rightarrow Problem \mathcal{R} feasible (converse, not true in general)
- Solution blocks of relaxed problem, not rank-one in general
- *Randomization*: generate candidate beamforming vectors
- Ensure constraints of Problem $\mathcal{I} \Rightarrow$ solve \mathcal{MGPC}
- LP ☺

$$a_{k,i} := |\mathbf{w}_k^H \mathbf{h}_i|^2$$

$$\beta_k := \|\mathbf{w}_k\|_2^2$$

p_k power boost factor
for multicast group k

\mathcal{MGPC} :

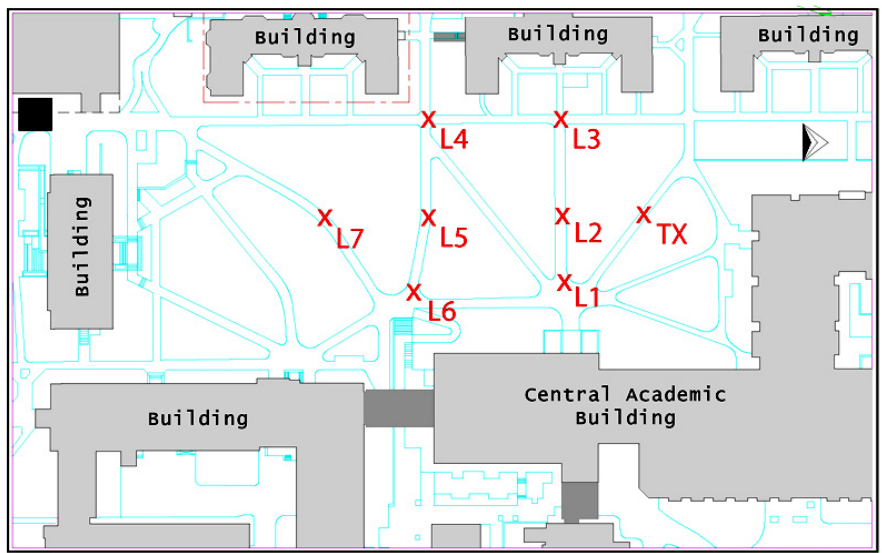
$$\min_{\{p_k \in \mathbb{R}\}_{k=1}^G} \sum_{k=1}^G \beta_k p_k$$

$$\text{s.t. : } \frac{p_k a_{k,i}}{\sum_{l \neq k} p_l a_{l,i} + \sigma_i^2} \geq c_i,$$

$$\forall i \in \mathcal{G}_k, \forall k \in \{1, \dots, G\},$$

$$p_k \geq 0, \forall k \in \{1, \dots, G\}.$$

Experimental results



<http://www.ece.ualberta.ca/~mimo/>
Often optimal, despite relaxation;
Not far from optimal (3-4dB) in
most cases

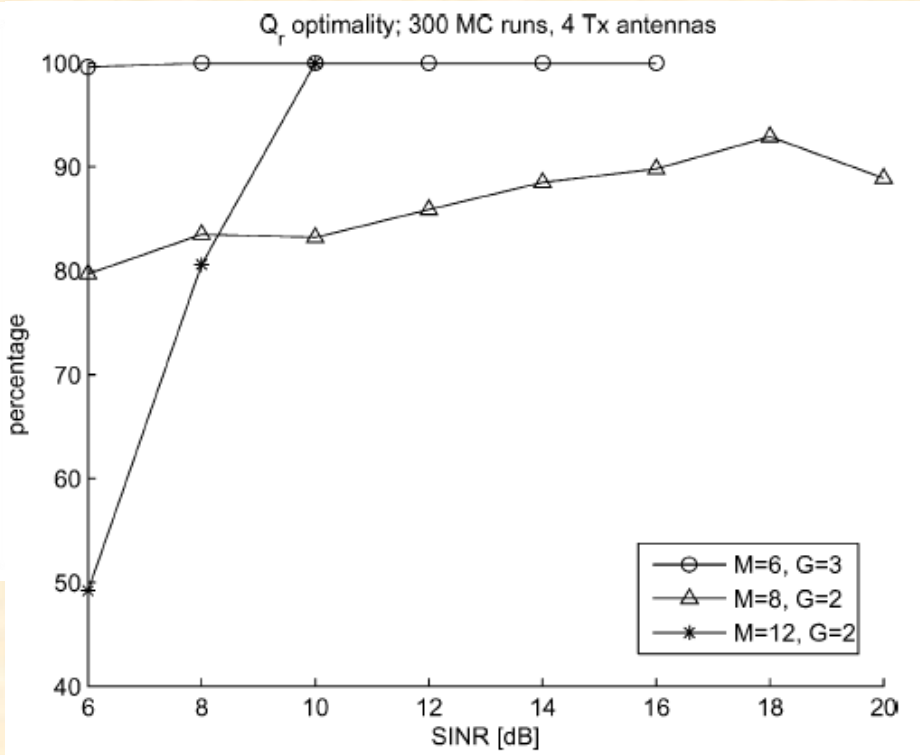


Fig. 4. Q_r optimality; 300 MC runs, 4 Tx antennas.

Analytical Approximation Performance Guarantees

Result (LuoSidTseZha SIOPT): Let v_o denote the optimum value of the single-group multicast beamforming problem (which is NP-hard), and v_r denote the optimum value of the associated SDR. Then, in the complex case,

$$v_r \leq v_o \leq 8Mv_r$$

(M is the total number of subscribing receivers) and Gaussian randomization with ~ 50 samples generates a feasible approximate solution that satisfies this bound with very high probability.

- (Usually pessimistic: $c \ll 8M$ often the case in practice)

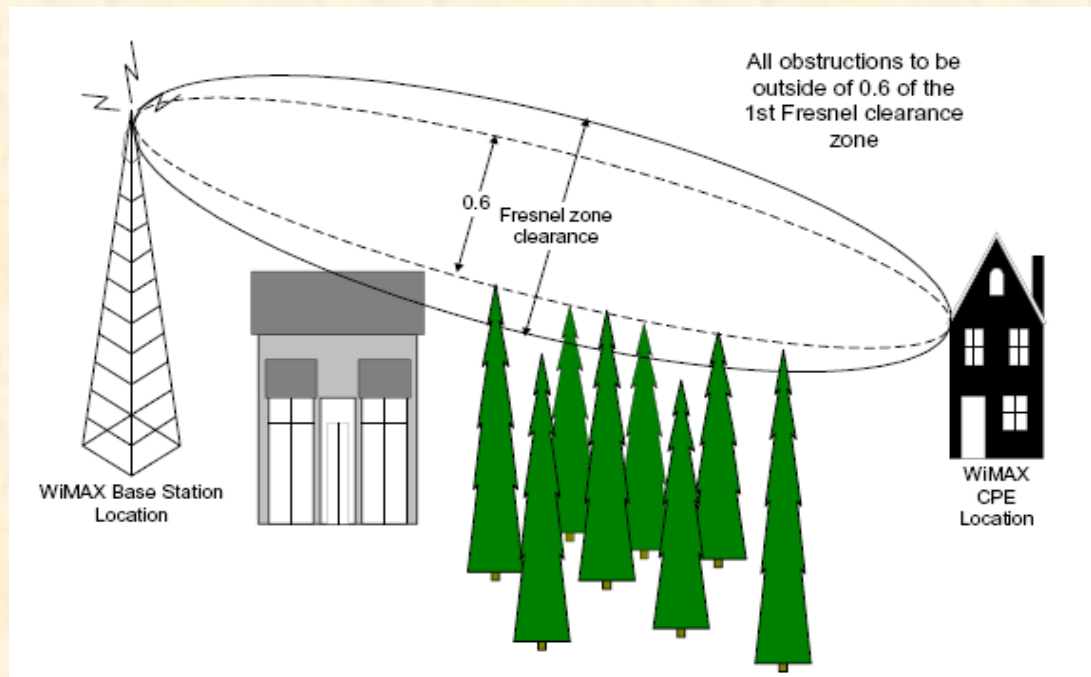
Max-min-fair version

$\mathcal{F} :$

$$\begin{aligned} & \max_{\{\mathbf{w}_k \in \mathbb{C}^N\}_{k=1}^G} \min_{k \in \{1, \dots, G\}} \min_{i \in \mathcal{G}_k} \frac{|\mathbf{w}_k^H \mathbf{h}_i|^2}{\sum_{\ell \neq k} |\mathbf{w}_\ell^H \mathbf{h}_i|^2 + \sigma_i^2} \\ \text{s.t. : } & \sum_{k=1}^G \|\mathbf{w}_k\|_2^2 \leq P. \end{aligned}$$

Exact Globally Optimal Solution in the Vandermonde Case (1)

- Motivation: fixed wireless LoS communications, e.g., WiMAX



Exact Globally Optimal Solution in the Vandermonde Case (2)

- For ULA, far-field / LoS (or, single-path) scenario
 \rightarrow Vandermonde channel vectors $\mathbf{h}_i = [1 \ e^{j\theta_i} \ e^{j2\theta_i} \ \dots \ e^{j(N-1)\theta_i}]^T$
- Numerical observation: SDR consistently rank-1!
Suggests: Problem not NP-hard, in fact convex in this case?
- Rx signal power at user i from beam k : $|\mathbf{w}_k^H \mathbf{h}_i|^2 = \sum_{\ell=-(N-1)}^{N-1} r_k(\ell) e^{j\theta_i \ell}$
- Autocorrelation fun.: $r_k(\ell) = \sum_{m=1}^{N-\ell} w_k(m) w_k^*(m + \ell), \quad 0 < \ell \leq N - 1$
- Conjugate-symmetric about the origin: $r_k^*(-\ell) = r_k(\ell)$

Exact Globally Optimal Solution in the Vandermonde Case (3)

- $\mathbf{r}_k := [r_k(-N+1), \dots, r_k(-1), r_k(0), r_k(1), \dots, r_k(N+1)]^T$,
 $\mathbf{f}_i := [e^{-j\theta_i(N-1)}, \dots, e^{-j\theta_i}, 1, e^{j\theta_i}, \dots, e^{j\theta_i(N-1)}]^T$
 $|\mathbf{w}_k^H \mathbf{h}_i|^2 = \mathbf{f}_i^T \mathbf{r}_k \quad r_k(0) = r_k(N) = \sum_{m=1}^N w_k(m) w_k^*(m) = \|\mathbf{w}_k\|_2^2$
- Equivalent reformulation:

$$\min_{\{\mathbf{r}_k\}_{k=1}^G} \sum_{k=1}^G \mathbf{r}_k(N)$$

$$\text{s.t. : } \mathbf{f}_i^T \mathbf{r}_k \geq \gamma_i \sum_{\ell \neq k} \mathbf{f}_i^T \mathbf{r}_\ell + \gamma_i \sigma_i^2, \quad \forall i \in \mathcal{G}_k, \quad \forall k \in \{1, \dots, G\},$$

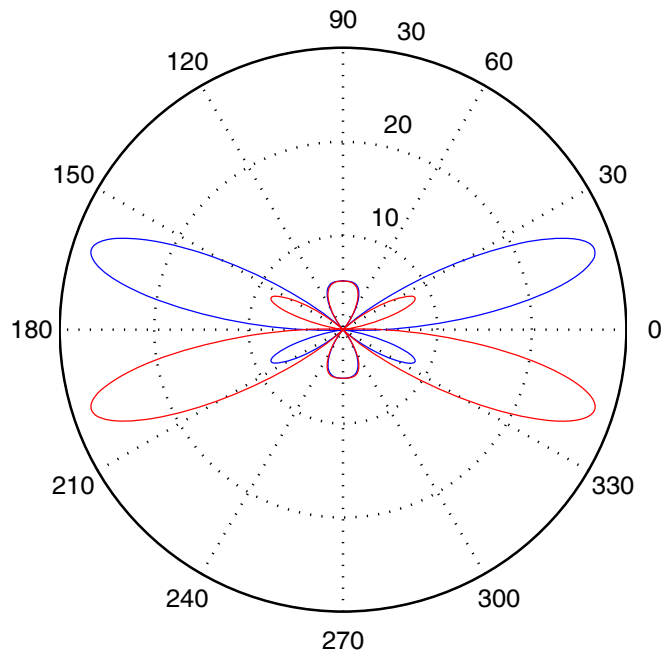
\mathbf{r}_k : autocorrelation vector, $\forall k \in \{1, \dots, G\}$

- Autocorrelation constraints equivalent to LMIs [AlkVan02] \rightarrow SDP
- ACS \rightarrow spectral factorization \rightarrow optimal beamvectors

Example

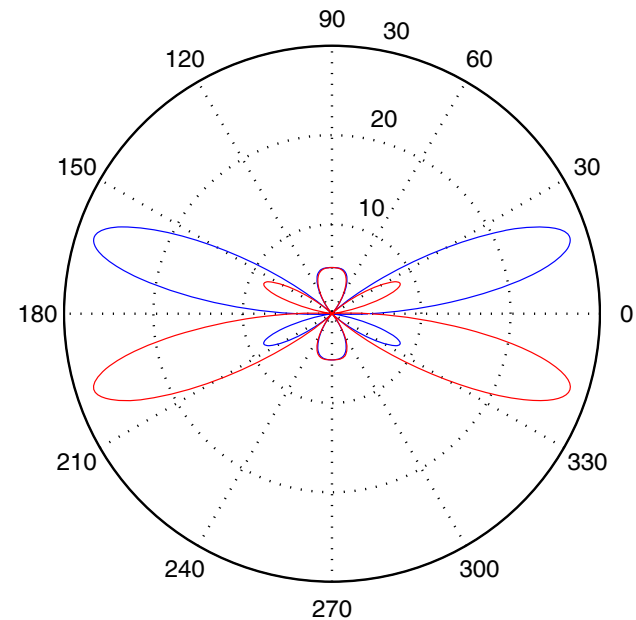
- $N = 4, \sigma_i^2 = \sigma^2 = 1, \gamma_i = \gamma = 10\text{dB}$

Algorithm 1: SDR + Randomization + MGPC



24 users in 2 groups, spaced 10 deg apart

Algorithm 2: SDP + Spectral factorization



24 users in 2 groups, spaced 10 deg apart

Robust Multicast Beamforming for imperfect CSI

- Perfect CSI: $\tilde{\mathbf{h}}_i := \mathbf{h}_i / \sqrt{\gamma_i \sigma_i^2}$

$$\begin{aligned} \text{ONRB :} \\ \min_{\mathbf{w} \in \mathbb{C}^N} \|\mathbf{w}\|_2^2 \\ \text{s.t. :} \quad |\mathbf{w}^H \tilde{\mathbf{h}}_i|^2 \geq 1, \quad \forall i \in \{1, \dots, M\}. \end{aligned}$$

- Robust version for imperfect CSI:

$$\begin{aligned} \text{RB :} \\ \min_{\mathbf{w} \in \mathbb{C}^N} \|\mathbf{w}\|_2^2 \\ \text{s.t. :} \quad |\mathbf{w}^H \tilde{\mathbf{h}}_i|^2 \geq 1, \quad \forall \tilde{\mathbf{h}}_i \in \mathcal{B}_\epsilon(\bar{\mathbf{h}}_i), \quad \forall i \in \{1, \dots, M\}. \end{aligned}$$

Robust Multicast Beamforming

- Result: Let \mathbf{w}' be an exact solution of \mathcal{RB} .
Then $\mathbf{w}'/(1 + \epsilon\|\mathbf{w}'\|)$ is an exact solution of ONRB.
Conversely, if \mathbf{w}_o is an exact solution of ONRB,
then $\mathbf{w}_o/(1 - \epsilon\|\mathbf{w}_o\|)$ is an exact solution of \mathcal{RB} .

Multicast Beamforming: Recap

- Multi-group multicast transmit beamforming under SINR constraints is NP-hard in general [KarSidLuo,SidDavLuo]
- Good & efficient approximation algorithms via SDR
- In the important special case of Vandermonde steering vectors it is in fact SDP – can be solved exactly & efficiently!
- For general steering vectors, exact solutions of the robust and non-robust versions of the single-group (broadcast) problem related via simple one-to-one scaling transformation!
- For Vandermonde steering vectors, robust version of the multi-group multicast problem is convex as well! [KarSidLuo]

Part II: Joint Multicast Beamforming and Admission Control

- Joint work w/ Vivi Matskani, Tom Luo, Leandros Tassiulas
- Inter-group interference and/or power constraint \rightarrow infeasibility \rightarrow admission control
- Joint multicast beamforming and admission control: MDR
- Single multicast group: important special case, in view of UMTS-LTE / E-MBMS
- MDR works for multiple co-channel multicast groups; will focus on single group for brevity
- In this case, infeasibility arises due to Tx power constraint

Infeasibility and Admission Control

- Often *infeasible* \rightarrow *admission control*
- Goal: max # users served @ pre-specified SNR levels for a given P .
- Two stages:

$$S_o = \operatorname{argmax}_{S \subseteq \mathcal{U}, \{\mathbf{w} \in \mathbb{C}^N\}} |S|$$

$$\text{subject to : } \|\mathbf{w}\|_2^2 \leq P,$$

$$\frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \geq c_i, \forall i \in S,$$

\rightarrow

$$\min_{\mathbf{w} \in \mathbb{C}^N} \|\mathbf{w}\|_2^2$$

$$\frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2} \geq c_i, \forall i \in S_o.$$

Single-stage reformulation

$$\min_{\mathbf{w} \in \mathbb{C}^N, \{s_i \in \{-1, +1\}\}_{i \in \mathcal{U}}} \epsilon \|\mathbf{w}\|_2^2 + (1 - \epsilon) \sum_{i \in \mathcal{U}} (s_i + 1)^2$$

$$\text{subject to : } \|\mathbf{w}\|_2^2 \leq P,$$

$$\frac{|\mathbf{w}^H \mathbf{h}_i|^2 + \delta^{-1} (s_i + 1)^2}{\sigma_i^2} \geq c_i, \forall i \in \mathcal{U}.$$

Result: (Cf. [MatSidLuoTas:07-08]) For $\delta \leq \min_{i \in \mathcal{U}} \frac{4c_i^{-1}}{\sigma_i^2}$

and $\epsilon < \frac{1}{P/4 + 1}$, the reformulated problem is always feasible, and its solution simultaneously maximizes the number of users served and minimizes the power required to serve them.

Getting ‘close’ to a convex problem

$\mathbf{W} := \mathbf{w}\mathbf{w}^H$, $\mathbf{S}_i := \mathbf{s}_i\mathbf{s}_i^T$, where $\mathbf{s}_i := [s_i \ 1]^T$, and
 $\mathbf{H}_i := \mathbf{h}_i\mathbf{h}_i^H$; rewrite as

$$\min_{\mathbf{W} \in \mathbb{C}^{N \times N}, \{\mathbf{S}_i \in \mathbb{R}^{2 \times 2}\}_{i \in \mathcal{U}}} \epsilon \text{Tr}(\mathbf{W}) + (1 - \epsilon) \sum_{i \in \mathcal{U}} \text{Tr}(\mathbf{1}_{2 \times 2} \mathbf{S}_i)$$

subject to : $\text{Tr}(\mathbf{W}) \leq P$,

$$\frac{\text{Tr}(\mathbf{H}_i \mathbf{W}) + \delta^{-1} \text{Tr}(\mathbf{1}_{2 \times 2} \mathbf{S}_i)}{\sigma_i^2} \geq c_i, \forall i \in \mathcal{U},$$

$$\mathbf{W} \geq 0, \text{rank}(\mathbf{W}) = 1,$$

$$\mathbf{S}_i \geq 0, \text{rank}(\mathbf{S}_i) = 1, \mathbf{S}_i(1, 1) = \mathbf{S}_i(2, 2) = 1, \forall i \in \mathcal{U}.$$

Semidefinite Relaxation (SDR)

Drop rank-1 constraints \rightarrow convex

$$\min_{\mathbf{W} \in \mathbb{C}^{N \times N}, \{\mathbf{S}_i \in \mathbb{R}^{2 \times 2}\}_{i \in \mathcal{U}}} \epsilon \text{Tr}(\mathbf{W}) + (1 - \epsilon) \sum_{i \in \mathcal{U}} \text{Tr}(\mathbf{1}_{2 \times 2} \mathbf{S}_i)$$

$$\text{subject to : } \text{Tr}(\mathbf{W}) \leq P,$$

$$\frac{\text{Tr}(\mathbf{H}_i \mathbf{W}) + \delta^{-1} \text{Tr}(\mathbf{1}_{2 \times 2} \mathbf{S}_i)}{\sigma_i^2} \geq c_i, \quad \forall i \in \mathcal{U},$$

$$\mathbf{W} \geq 0,$$

$$\mathbf{S}_i \geq 0, \mathbf{S}_i(1, 1) = \mathbf{S}_i(2, 2) = 1, \quad \forall i \in \mathcal{U}.$$

MDR- Algorithm

1. $\mathcal{U} \leftarrow \{1, \dots, K\}$
2. Solve the relaxed problem and let $\tilde{\mathbf{W}}$ denote the resulting transmit covariance matrix
3. $\tilde{\mathbf{w}}$ = principal component of $\tilde{\mathbf{W}}$, scaled to power $Tr(\tilde{\mathbf{W}})$.
4. For each $i \in \mathcal{U}$, check whether

$$\frac{|\tilde{\mathbf{w}}^H \mathbf{h}_i|^2}{\sigma_i^2} \geq c_i.$$

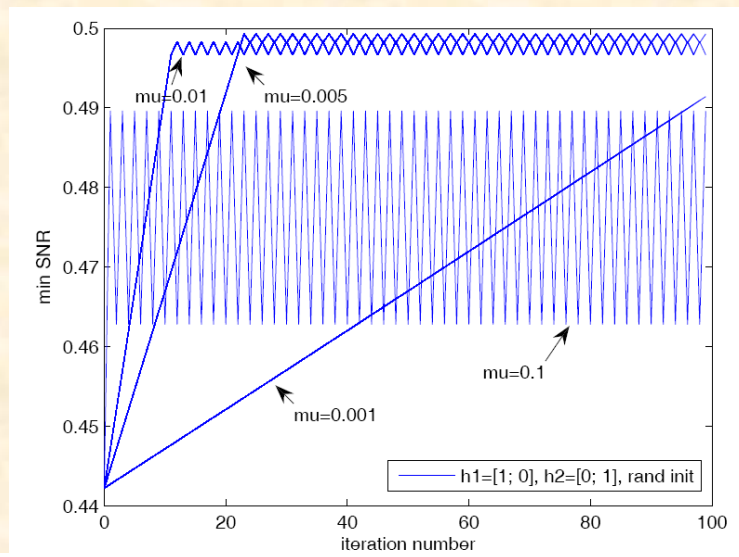
If true for all $i \in \mathcal{U}$, stop (a feasible solution has been found); else pick the user with largest gap to its target SNR, remove from \mathcal{U} and go to step 2.

Lozano's Algorithm

1. Init: $\mathbf{w} = [1 \ 0 \ \dots \ 0]^T$
2. Compute $SNR_i(t-1) = P\mathbf{w}_{t-1}^H \mathbf{H}_i \mathbf{w}_{t-1}$, $\forall i \in \mathcal{U}$
3. Sort $SNR_i(t-1)$, $\forall i \in \mathcal{U}$.
4. Drop a fixed proportion of users with lowest attained SNR s.
5. Find weakest link among remaining ones ($\rightarrow k$)
6. Take step in its direction: $\mathbf{w}_t = \mathbf{w}_{t-1} + \mu \mathbf{H}_k \mathbf{w}_{t-1}$;
then $\mathbf{w}_t = \mathbf{w}_t / \|\mathbf{w}_t\|_2$
7. Repeat until no significant change in minimum SNR .

Issues w/ Lozano's algorithm

- Simple algorithm, but intricate convergence behavior
- No guidelines for choosing μ
- We show via toy counter-example:
 - May shut-off users completely (no chance of admission) – fairness issue
 - May not converge
 - Can exhibit limit cycle behavior, even for very small μ



Example of
limit cycle behavior
of Lozano's algorithm.

$N = 2$ Tx, $K = 2$ users,
 $\mathbf{h}_1 = [1 \ 0]^T$, $\mathbf{h}_2 = [0 \ 1]^T$
 $\sigma_1 = \sigma_2 = P = 1$.
Random initialization,
 $\mu = 0.1$.

Proposed improvement - I: LLI

- [Lopez:2004]: Max average SNR beamformer \leftrightarrow principal component:

$$\mathbf{H} := [\mathbf{h}_1, \dots, \mathbf{h}_K] \ (N \times K);$$

$$[\mathbf{U}, \mathbf{S}, \mathbf{V}] = \text{svd}(\mathbf{H}^H); \mathbf{w}_0 = \mathbf{V}(:, 1);$$

- Use this for initialization
- PC can be tracked, e.g., using power method \rightarrow overall solution remains simple, adaptive
- **LLI**: Lozano with Lopez Initialization

Proposed improvement - II: dLLI

- Simple and effective way to suppress limit cycle behavior: damp μ according to a predefined back-off schedule. This should be balanced against our primary objective, which is to find a good solution
- The weight update of Lozano's algorithm can be interpreted as taking a step in the direction of the *local* subgradient of $\min_{i \in \mathcal{A}_t} SNR_i(t-1)$, where the minimum is taken over the currently active user set \mathcal{A}_t .

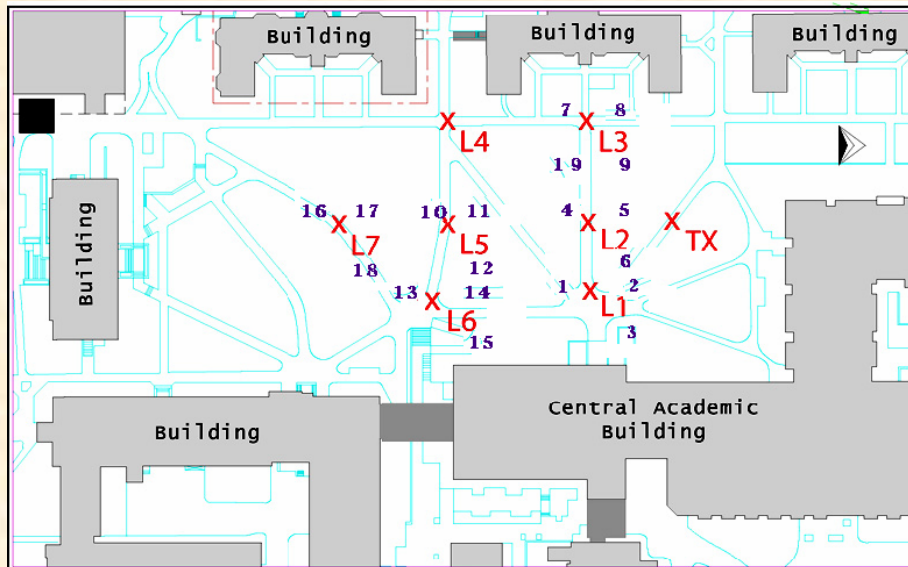
Proposed improvement - II: dLLI

- There are two difficulties here: this is not a subgradient in the usual sense, because it is only a local, not a global under-estimator of $\min_{i \in \mathcal{A}_t} SNR_i(t-1)$; and \mathcal{A}_t can change as iterations progress. These difficulties arise because we are dealing with a non-convex and NP-hard problem.
- dLLI: $\mu_t = \frac{\mu_{t-1}}{t/10}$ if $t \bmod 10 = 0$, else $\mu_t = \mu_{t-1}$ (back-off every 10 iterations).

Fair comparison

- MDR fixes min SNR, attempts to optimize coverage
- Lozano and (d)LLI fix coverage, attempt to optimize min SNR
- Proper comparison: min SNR vs. coverage operating characteristic (similar to ROC)
- Using measured channel data
- Benchmark: enumeration over all subsets; for each use SDR
- Per-subset problem is still NP-hard, but
 - enumeration+SDR ('ENUM') yields upper bound on min SNR (attainable performance)
 - when SDR returns rank-1 solution for maximal subset, it is overall optimal; this happens in vast majority of cases considered → ENUM yields tight upper bound

Measured channel data



<http://www.ece.ualberta.ca/~mimo/>

$N = 4$ Tx antennas

Left: Outdoor

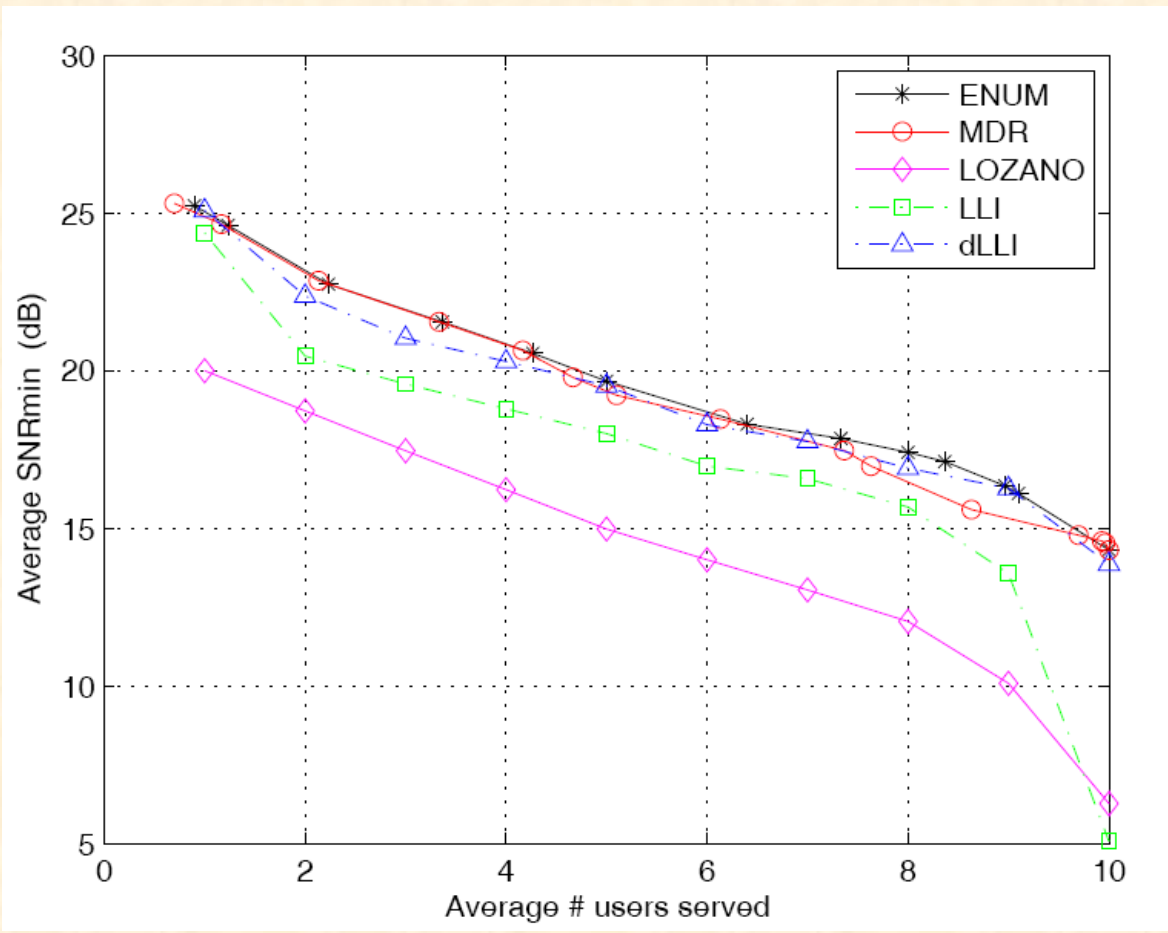
Right: Indoor

Results – I: Outdoor, I-CSIT

Instantaneous CSI-T.
30 Measured channel
snapshots.

$K = 10$ users, $P = 30$,
 $N = 4$ Tx, $c_i = c$,
 $\sigma_i^2 = \sigma^2 = 1, \forall i$.
MDR: $\epsilon = 10^{-10}$,
 $\delta < \min_{i \in \mathcal{U}} 4c^{-1}/\sigma_i^2$.
Lozano's, LLI: $\mu = 10^{-2}$,
accuracy (convergence)
 $= 10^{-3}$.

Complexity (per problem instance):
ENUM: about 2 min
MDR: between 10^{-2} and 1 sec
Lozano's, LLI: between 10^{-2} and 10^{-1} sec.



Results – II: Outdoor, LT-CSIT

Long-term CSI-T.

$K = 10$ users, $P = 30$,

$N = 4$ Tx, $c_i = c$,

$\sigma_i^2 = \sigma^2 = 1, \forall i$.

MDR: $\epsilon = 10^{-10}$,

$\delta < \min_{i \in \mathcal{U}} 4c^{-1}/\sigma_i^2$.

Lozano's, LLI: $\mu = 10^{-2}$,

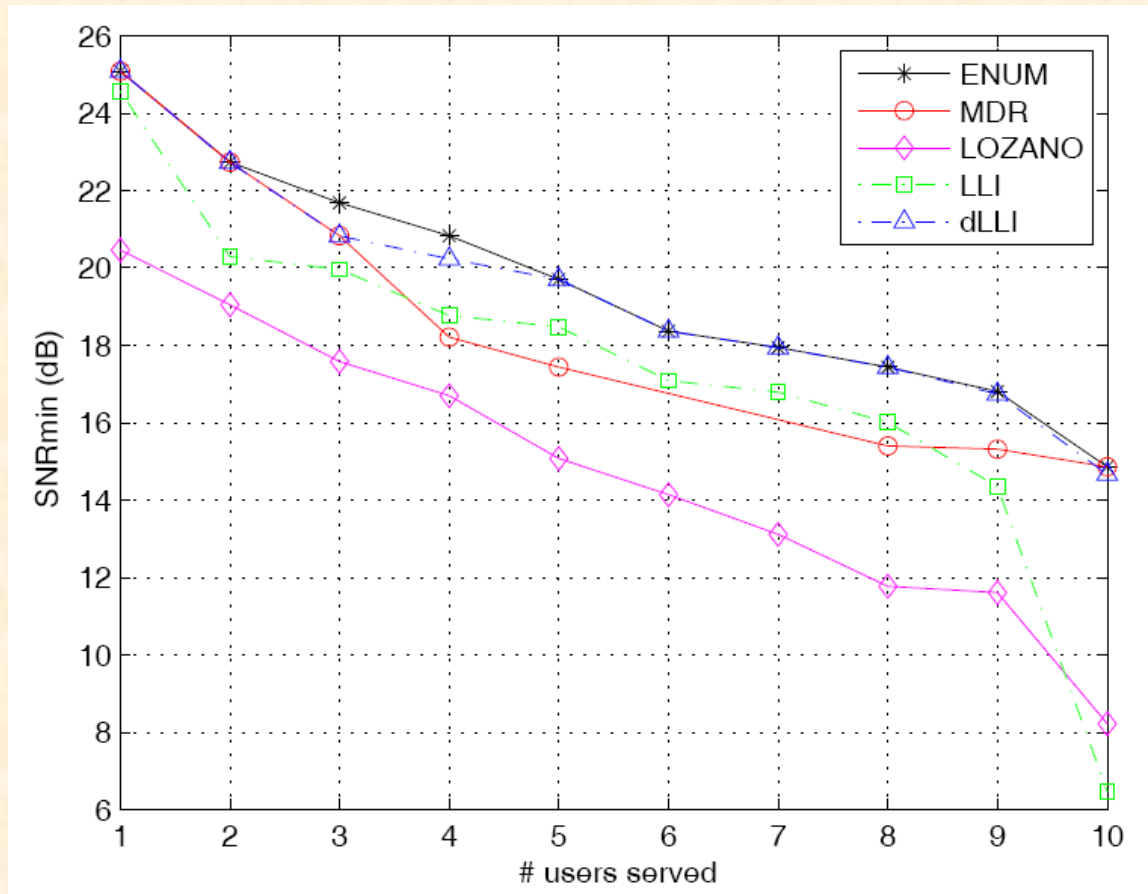
accuracy (convergence)
 $= 10^{-3}$.

Complexity:

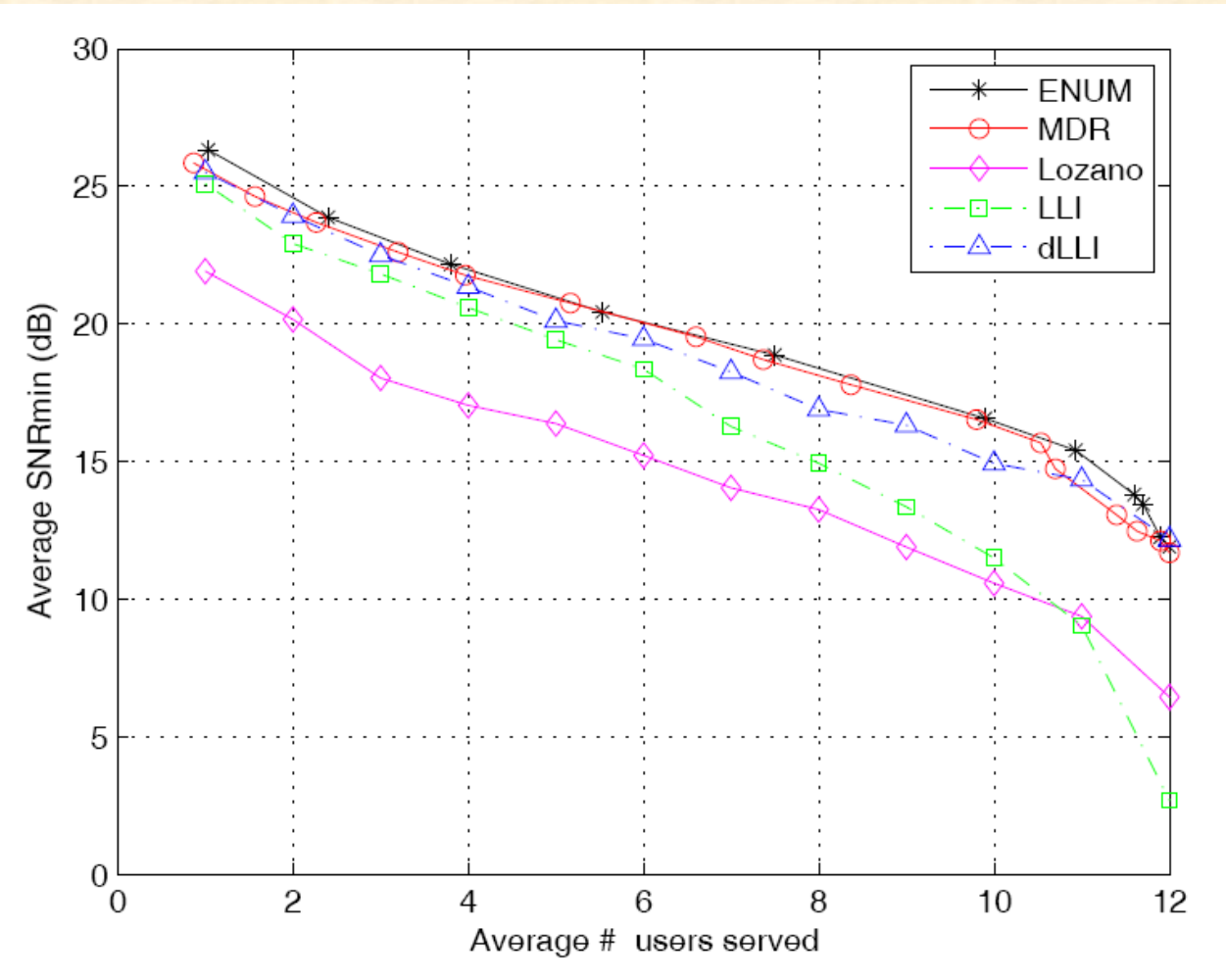
ENUM: about 2 min

MDR: between 10^{-2} and 1 sec

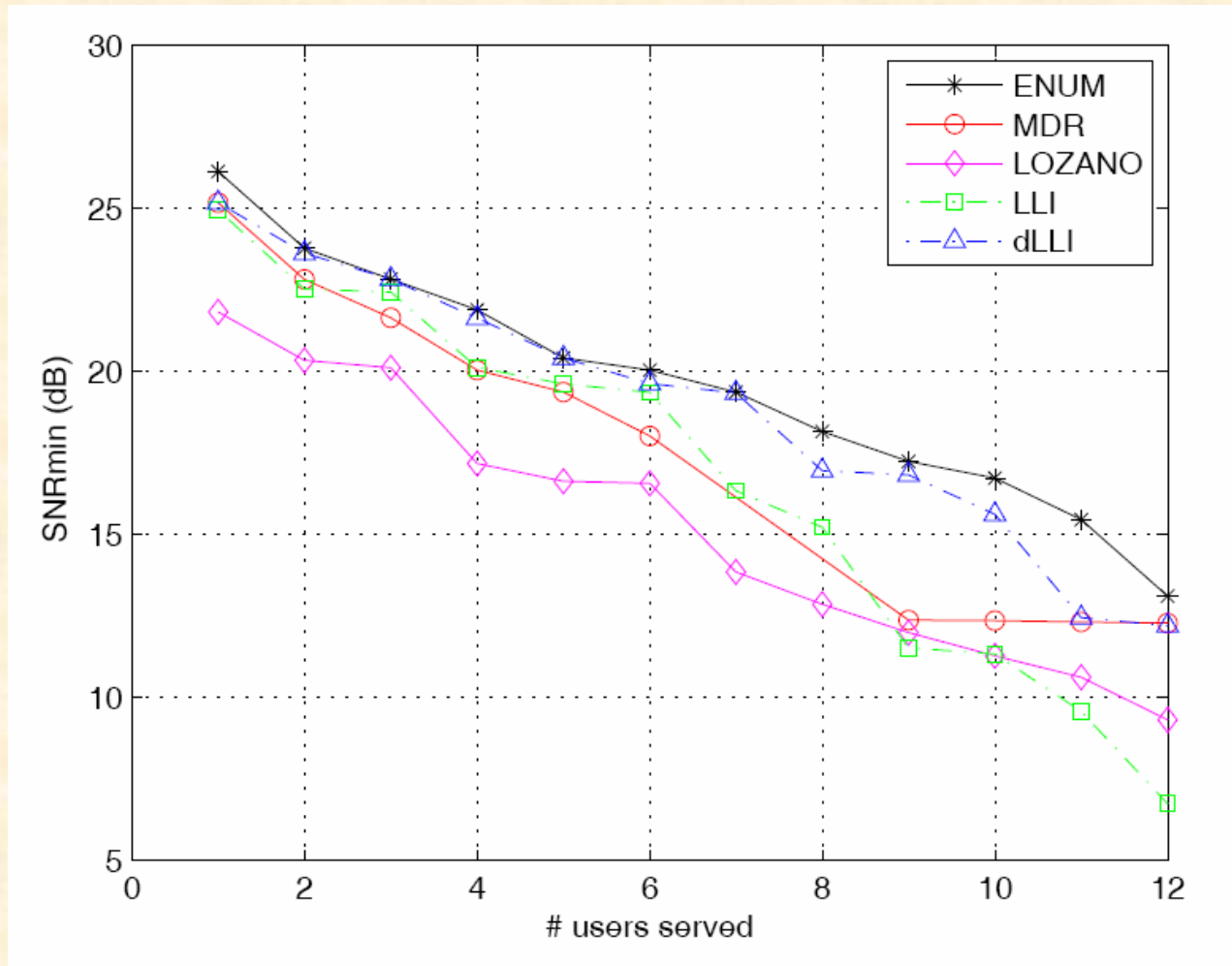
Lozano's, LLI: between 10^{-2} and 10^{-1} sec.



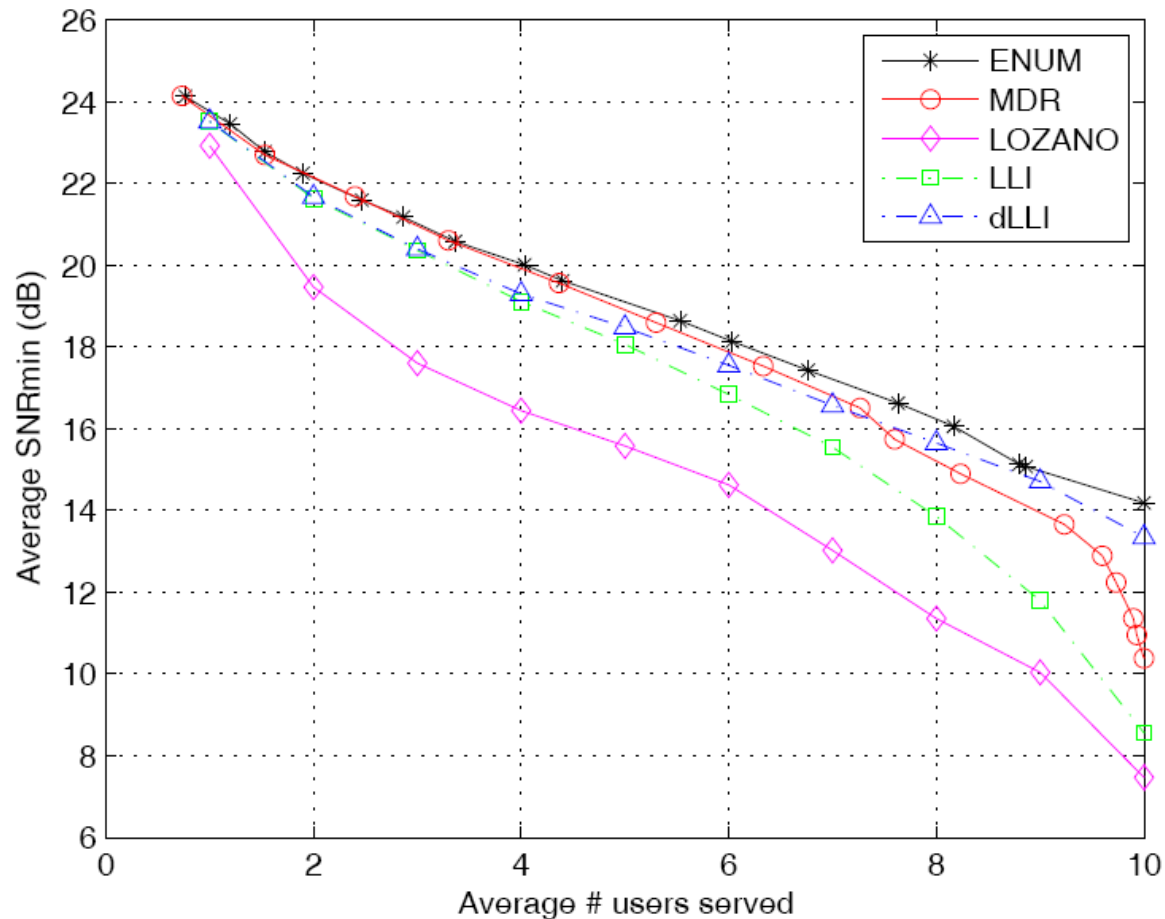
Results – III: Indoor, I-CSIT



Results – IV: Indoor, LT-CSIT



Results – V: iid Rayleigh, I-CSIT



Conclusions

- ENUM returned rank-1 solutions in all cases except full coverage; complexity exponential in K ; prohibitive for large K .
- dLLI and MDR emerge as clear winners
- dLLI best for LT-CSIT
- MDR best in certain I-CSIT cases
- dLLI is simpler and faster than MDR
- ... but MDR works for multiple groups
- Both close to optimal
- dLLI : significant improvement over Lozano's original algorithm; due to adaptive nature and only quadratic complexity → **ideal candidate for practical implementation in LTE / E-MBMS**

Sneak preview:

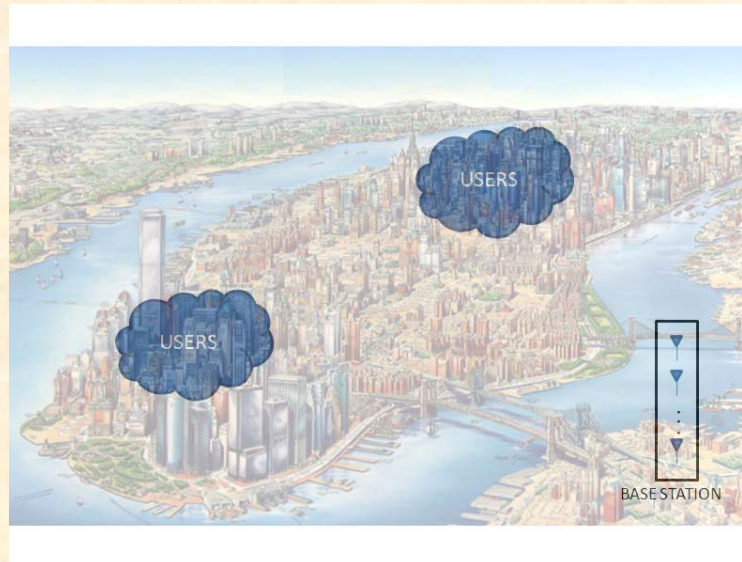
Multicast beamforming for minimum outage (Ntranos, Sidiropoulos, Tassiulas, IEEE TWC)

- Assume channel vectors random, drawn from, say, Gaussian
- Max # customers served under power constraint is NP-hard, even if you know their channels exactly.
- For large # customers, can approx. max # served by min $P(\text{outage})$
- Trivial for single Gaussian – and it doesn't require channel state – only channel statistics!
- NP-hard problem \rightarrow trivial one! ☺

Sneak preview:

Multicast beamforming for minimum outage (Ntranos, Sidiropoulos, Tassiulas, IEEE TWC)

- Promising ... but *Gaussian mixture* model is far more realistic for multicast



Sneak preview:

Multicast beamforming for minimum outage: Results

- When # kernels in mixture $>$ # Tx antennas, there's no escape from NP-hardness ... ☹
- But for 2-3 kernels (practical), optimal solution is tractable.
- For any number of kernels, effective approximation of very low computational complexity.
- Very interesting because approach requires no CSI, and still delivers (probabilistic) service guarantee
- Respects subscriber privacy concerns; requires no logging
- No reverse-link signaling

Thank you for your attention ☺